On ω-Regular Trace Languages

Classification and Synthesis

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Traces and trace languages

Traces as models of behaviors of distributed systems

- Traces model dependence/independence between actions
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Trace languages correspond closely with word languages

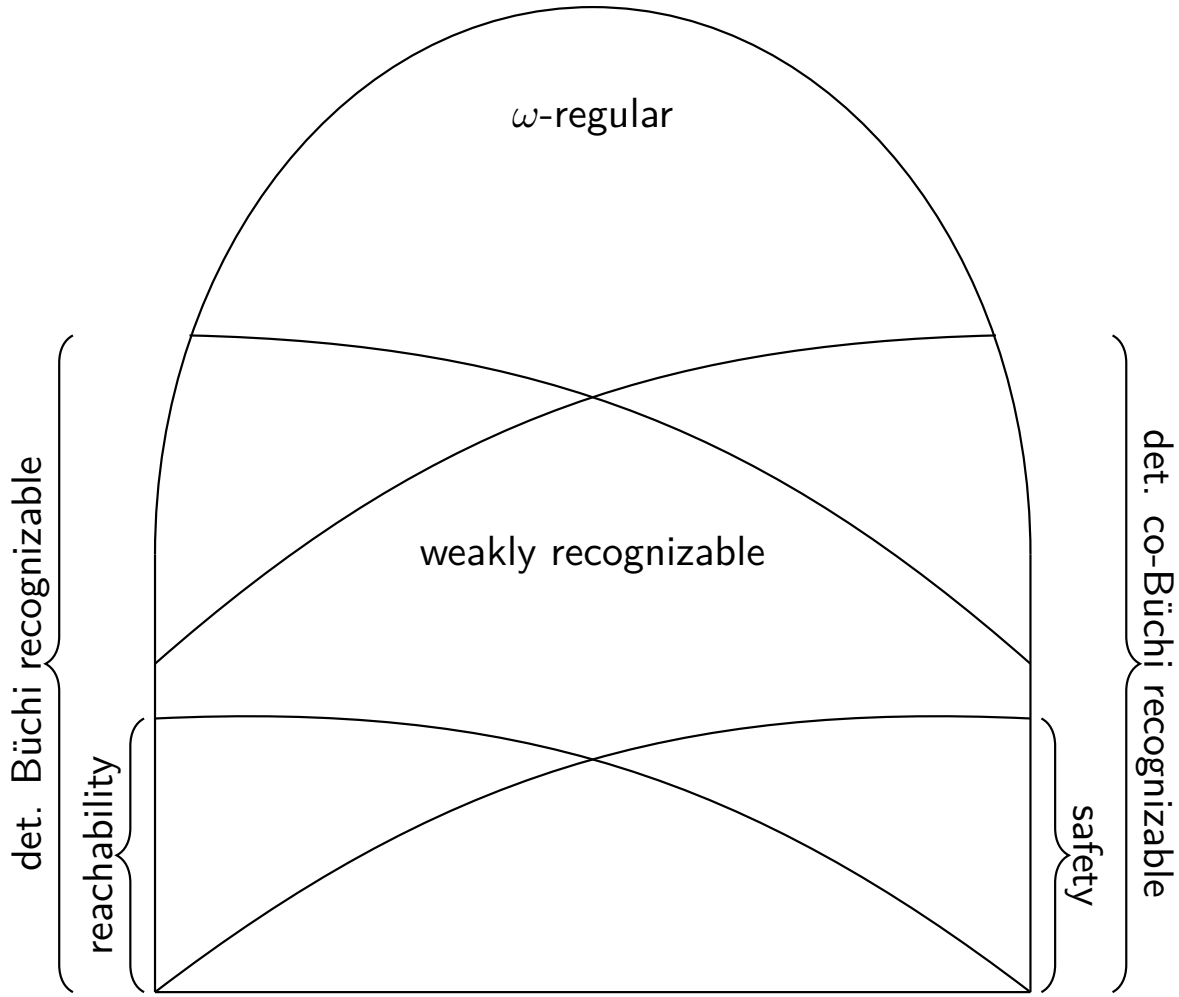
- A single **trace** represents an **equivalence class** of such partially commutative sequences
- A language of traces is **recognizable** if and only if the corresponding "trace-closed" word language is recognizable

End of similarities

Borel classification of ω-regular languages

Open problem

Can ω-regular trace languages also be classified in such a manner?



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1. Classification is good
2. Natural descriptions of ω-regular languages with the help of regular languages (using reachability, safety, and liveness conditions)
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4. Efficient algorithms for synthesis and verification for different classes
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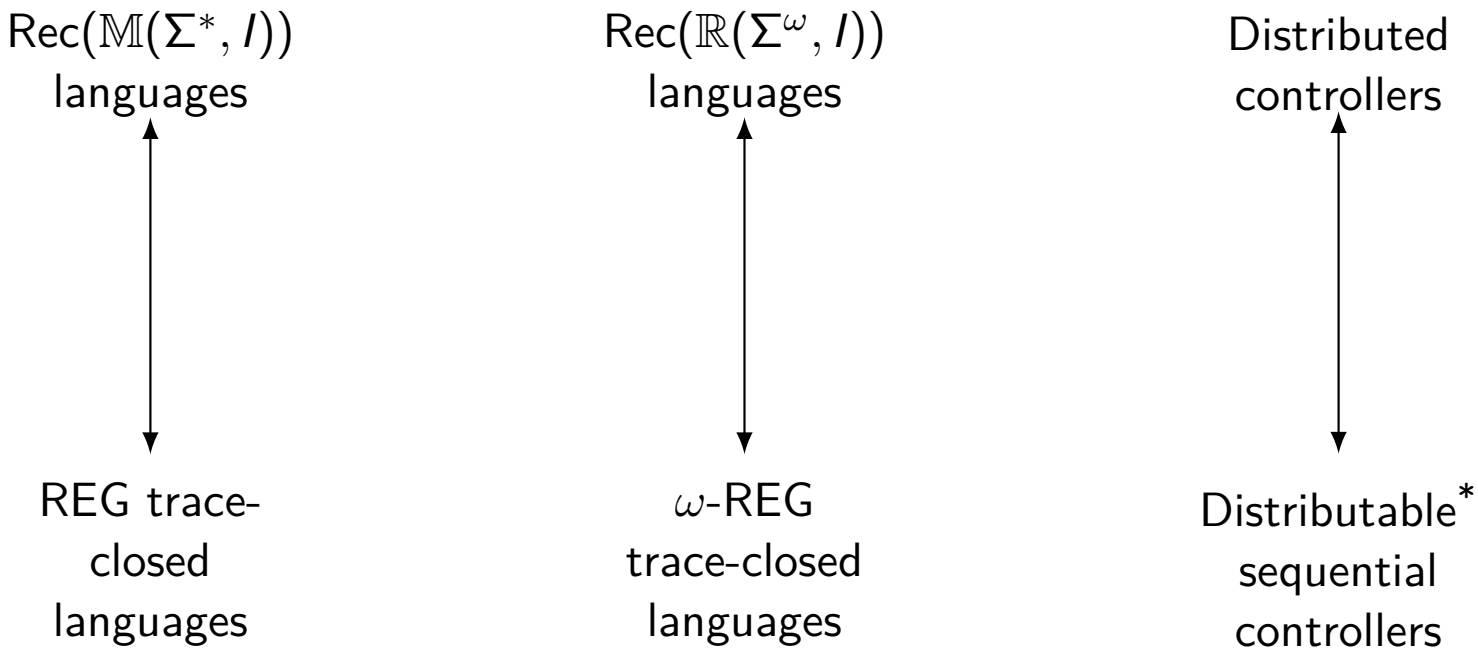

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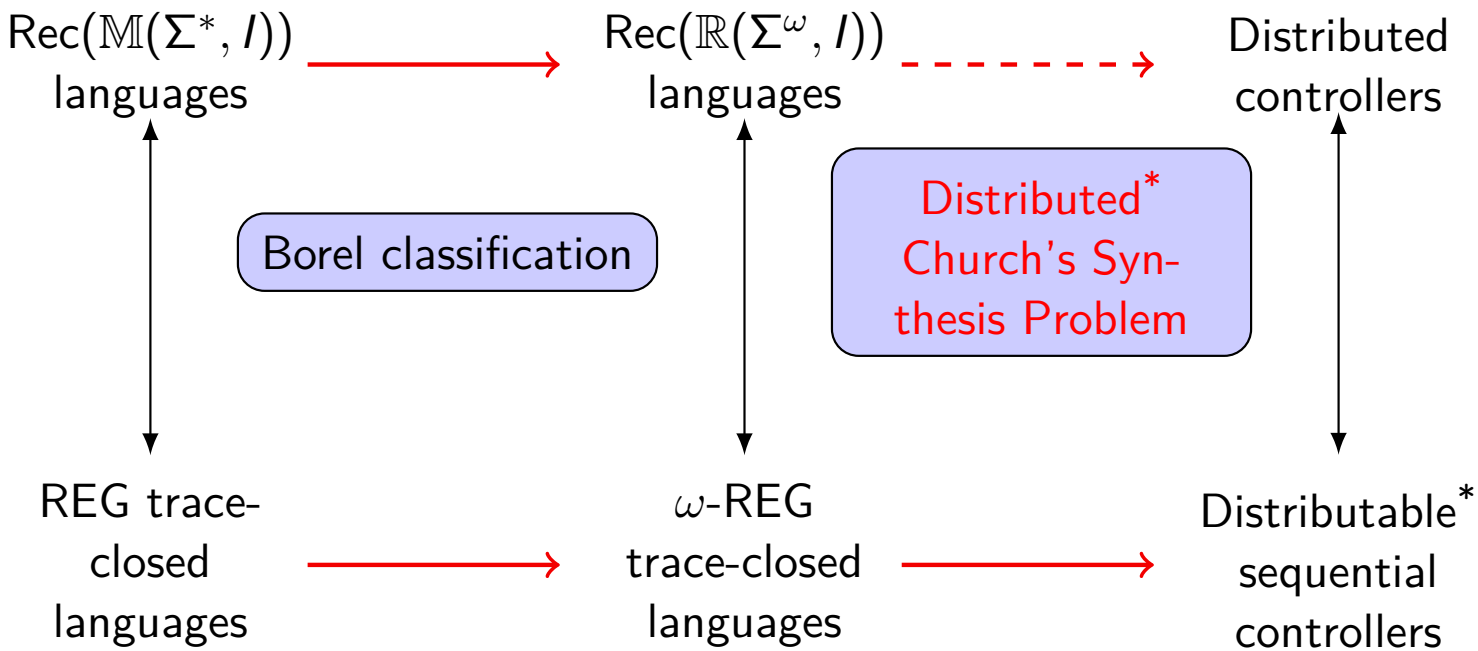

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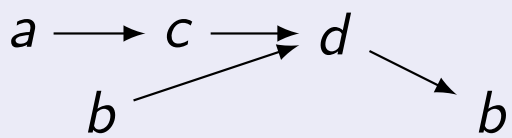

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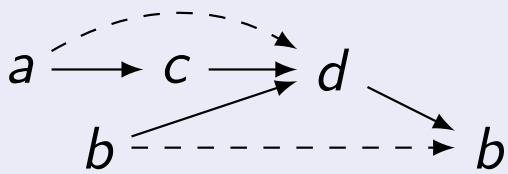

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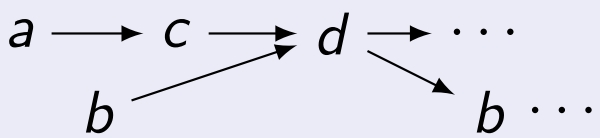

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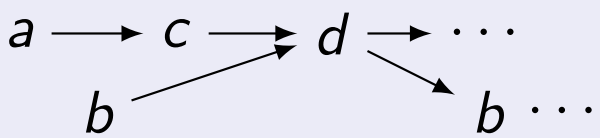
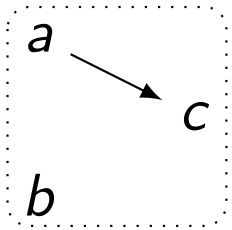
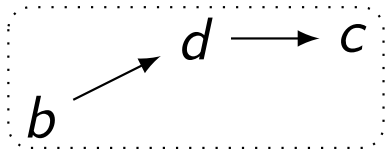


Assuming $a I b$ and $b I c$





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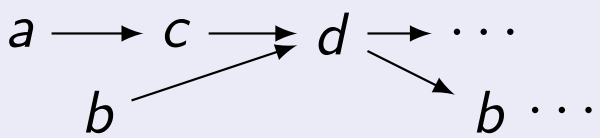

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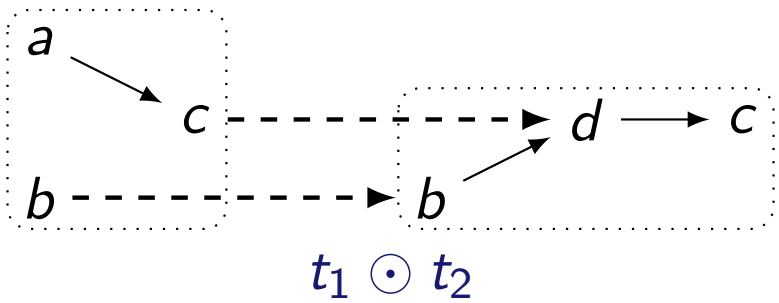

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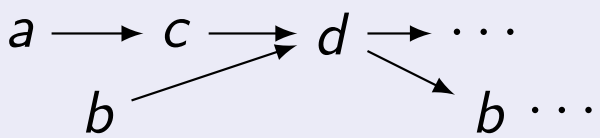

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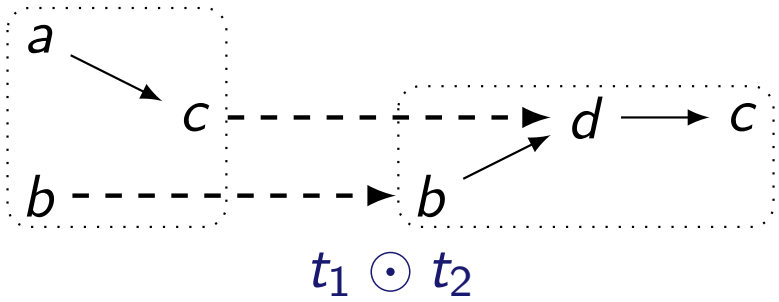

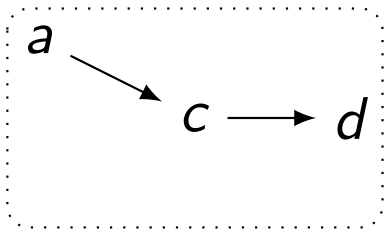

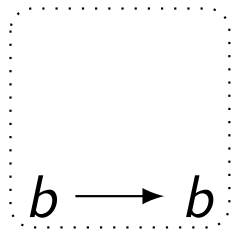

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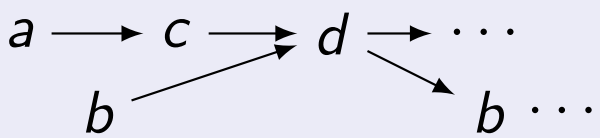

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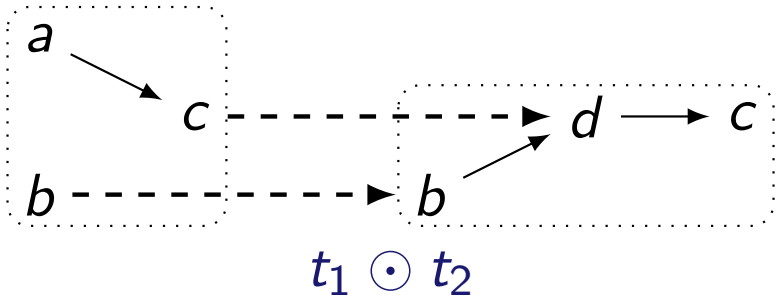
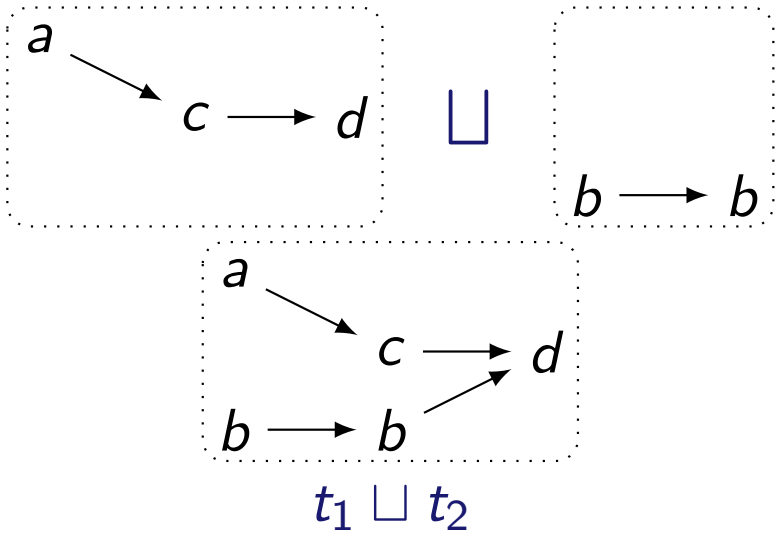


$t_1 \odot t_2$



$t_1 \sqcup t_2$

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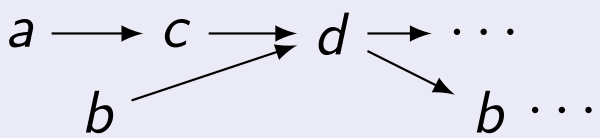

The set of all finite traces: $\mathbb{M}(\Sigma^*, I)$

The set of all infinite traces: $\mathbb{R}(\Sigma^\omega, I)$

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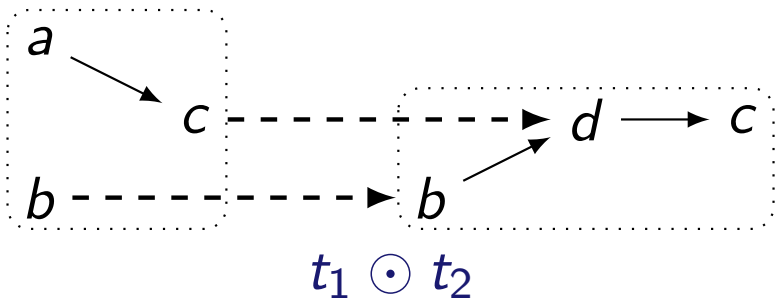

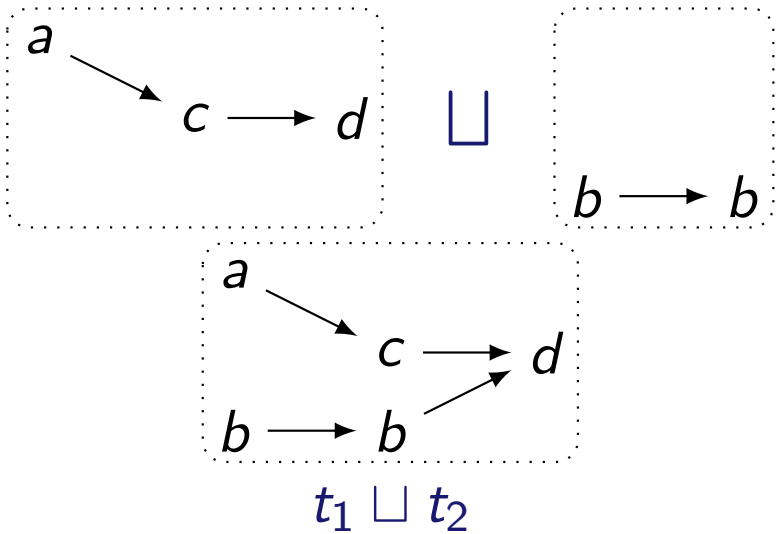

Preliminaries: Traces, words, languages

There is a tight connection between traces and words

- Mapping words to traces, $\Gamma : \Sigma^* \to \mathbb{M}(\Sigma^*, I)$

$$\Gamma(abcdb) = \quad \begin{array}{ccccccc} a & \longrightarrow & c & \longrightarrow & d & \searrow & \\ & b \nearrow & & & & & b \end{array}$$

- Mapping traces to trace-closed sets of words

$$\Gamma^{-1}\left(\begin{array}{ccccccc} a & \longrightarrow & c & \longrightarrow & d & \searrow & \\ & b \nearrow & & & & & b \end{array}\right) = \{bacdb, abcdb, acbdb\}$$

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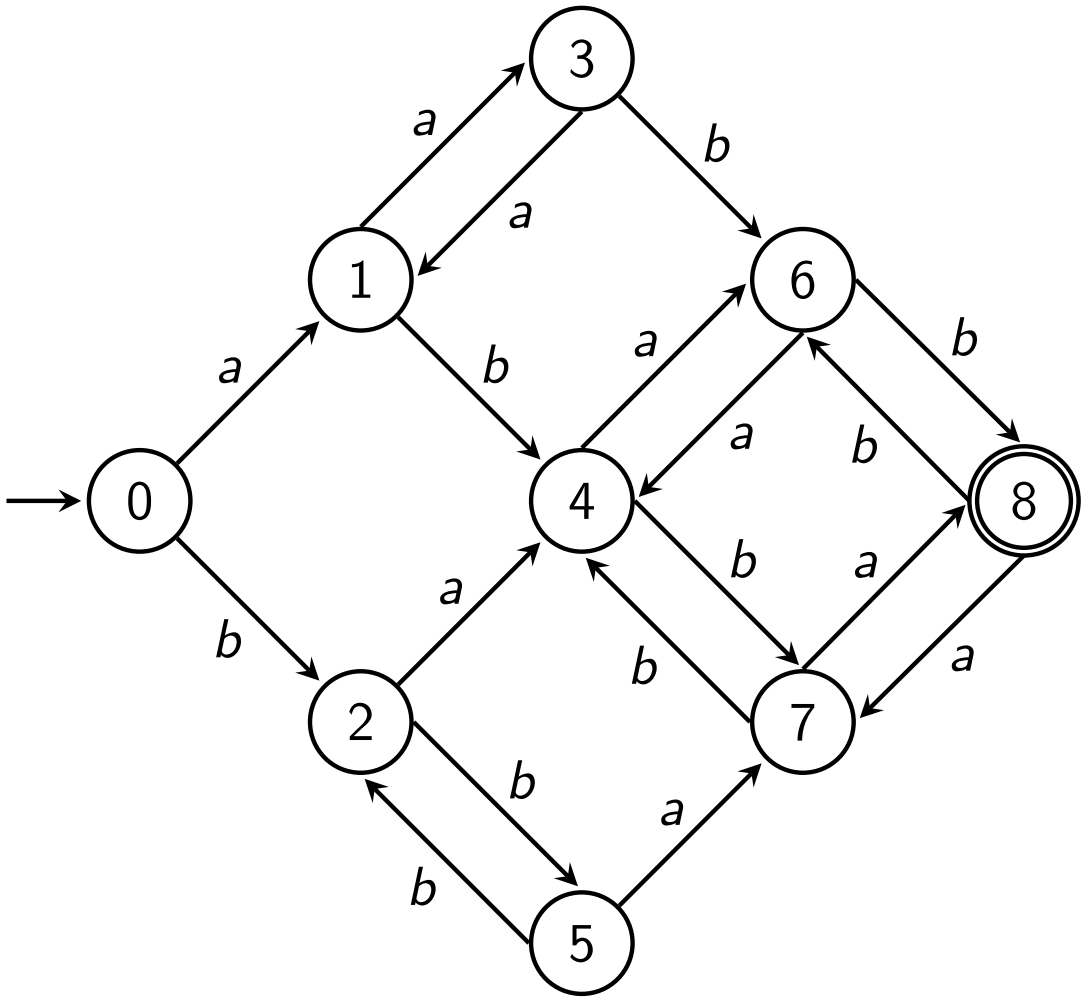
Definition

A trace language $T \subseteq \mathbb{M}(\Sigma^*, I)$ (resp. $\Theta \subseteq \mathbb{R}(\Sigma^\omega, I)$) is **recognizable** iff $\Gamma^{-1}(T)$ (resp. $\Gamma^{-1}(\Theta)$) is a recognizable word language.

Preliminaries: A final note on automata

Trace-closed languages are recognized by ***I*-diamond automata**

Let $\Sigma = \{a, b\}$, $a I b$. Define $K := [(aa)^+(bb)^+]_{\sim_I}$.



Infinitary extensions of regular trace languages

Definition

Let $T \in \mathrm{Rec}(\mathbb{M}(\Sigma^*, I))$. Its **infinitary extension** is the ω-trace language given by $\mathrm{ext}(T) := T \odot \mathbb{R}(\Sigma^\omega, I) = \bigcup_{t \in T} t \odot \mathbb{R}(\Sigma^\omega, I)$.

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$$\Gamma(ab(aabb)^\omega) = \begin{matrix} a \longrightarrow a \longrightarrow a \longrightarrow \cdots \\ b \longrightarrow b \longrightarrow b \longrightarrow \cdots \end{matrix} \in \mathrm{ext}(T)$$

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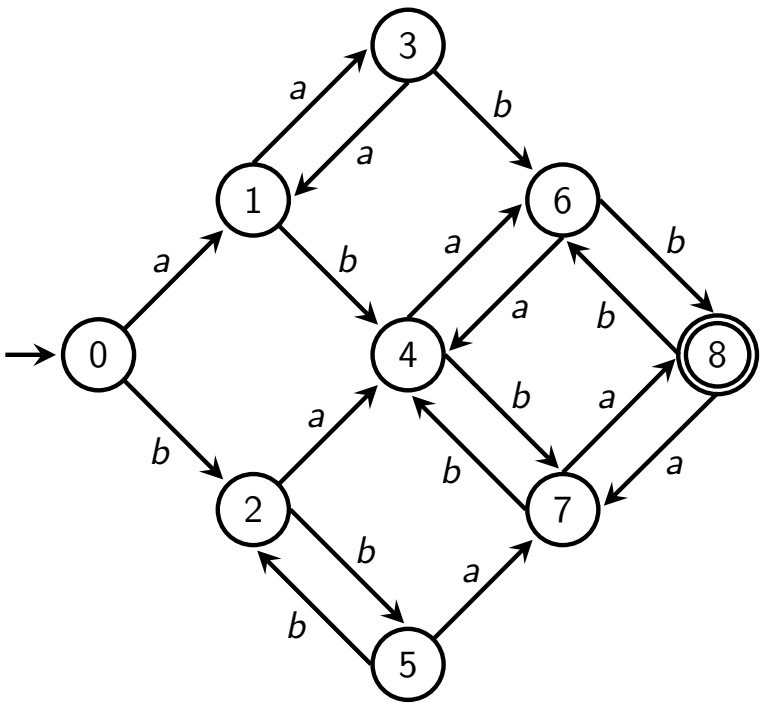
Ensuring ext(K) is trace-closed



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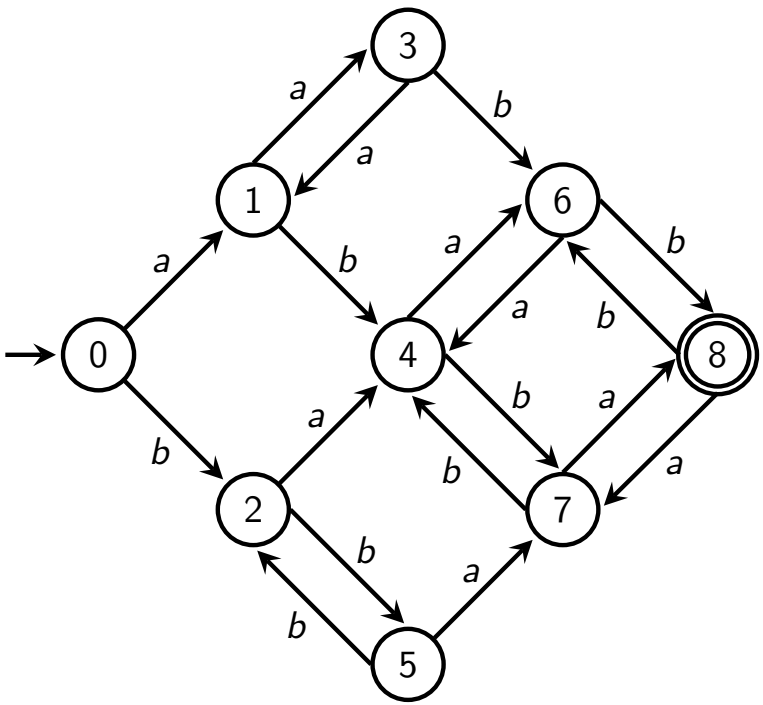

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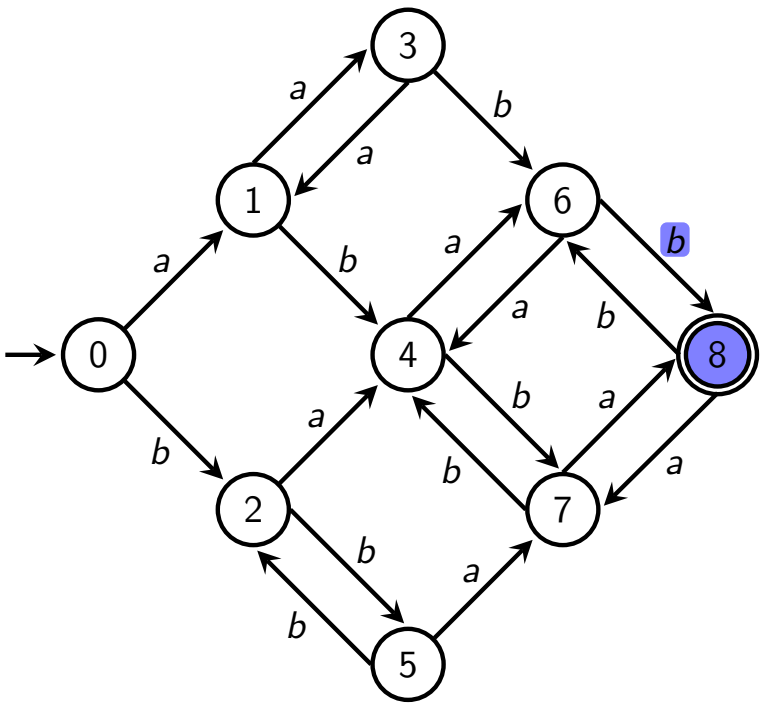

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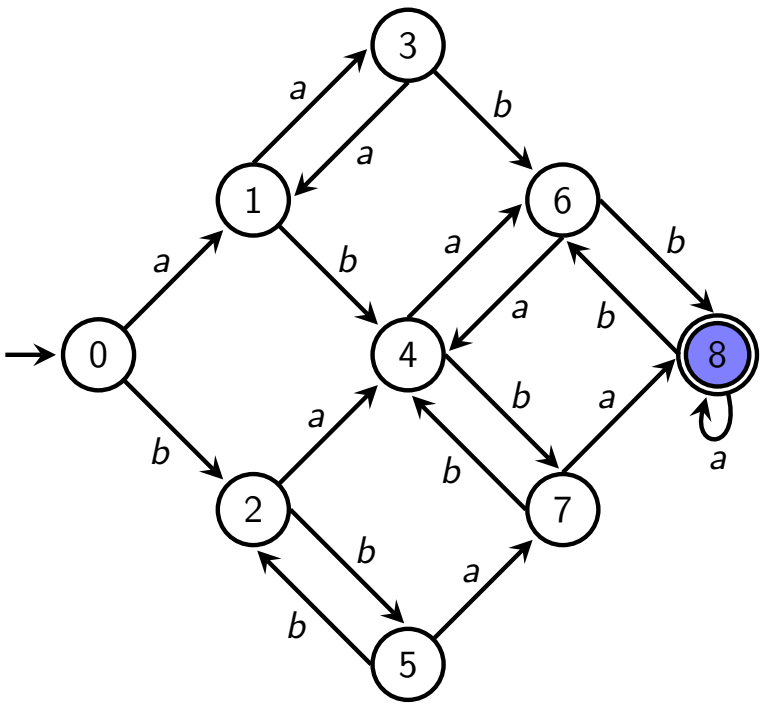

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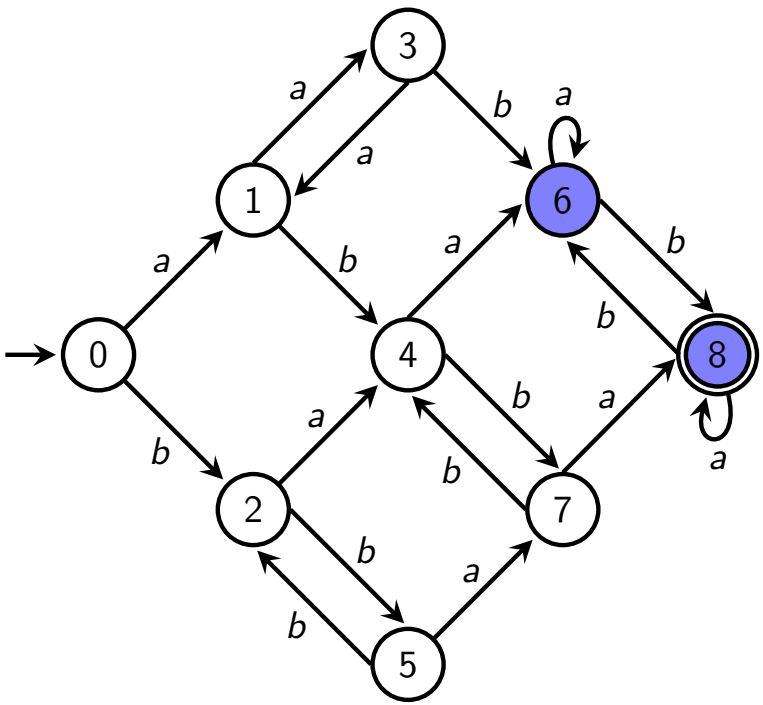

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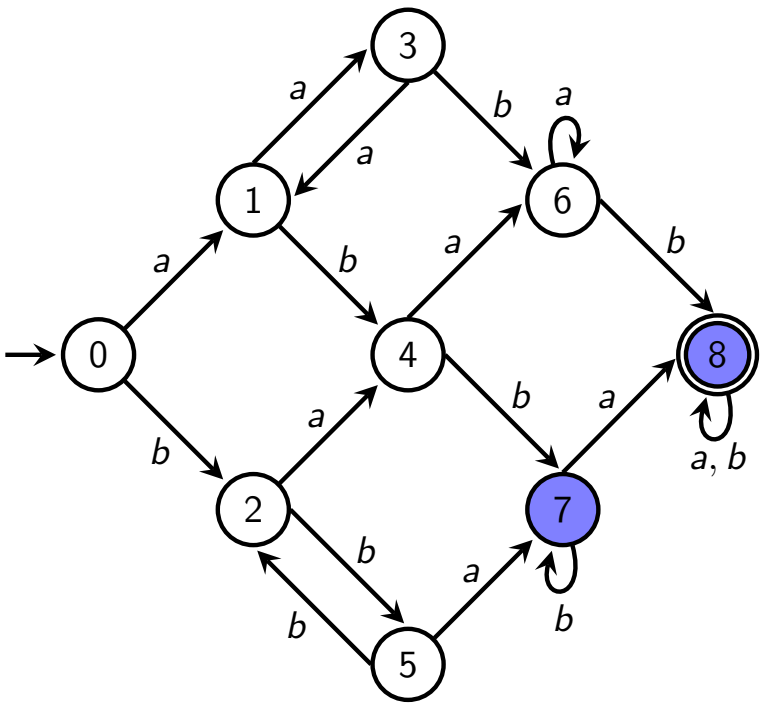

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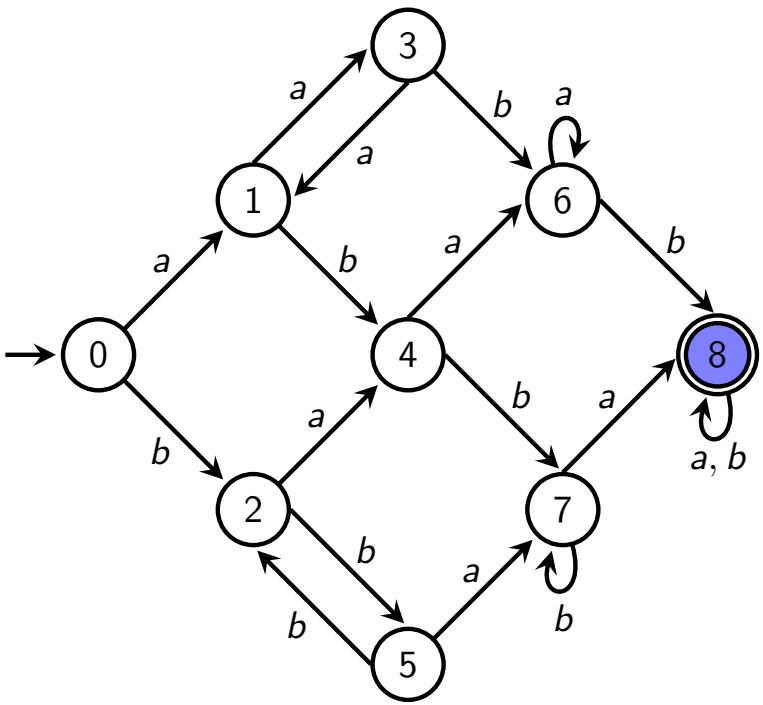

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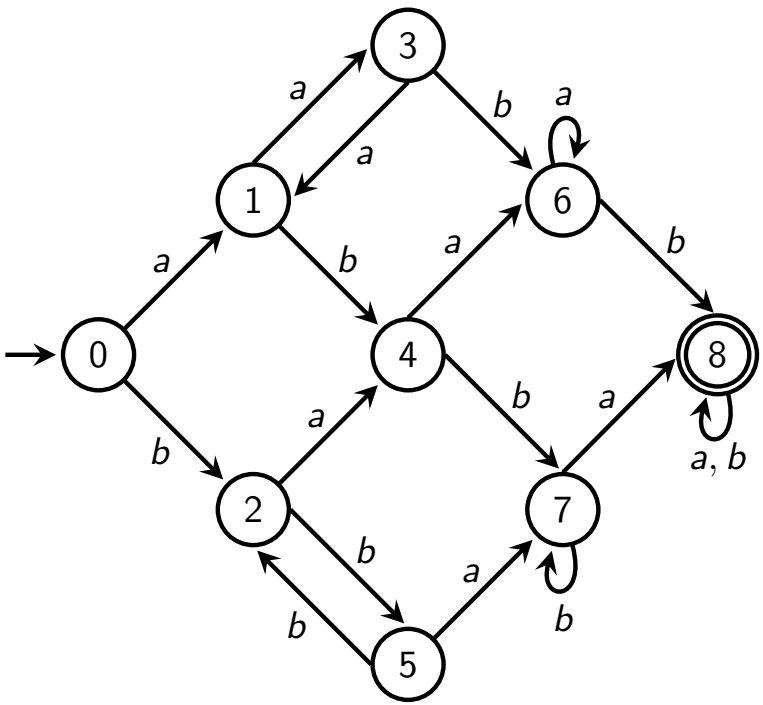
- $K_I = K_\Sigma$

Ensuring ext(K) is trace-closed

Definition (I-suffix extension)

For $K \in \mathrm{REG}$, trace-closed, define $K_I := K \cup \bigcup_{a \in \Sigma} [Ka^{-1}a \cdot I_a^*]_{\sim_I}$



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- $K_I = K_\Sigma$

I-suffix extension suffices

Lemma

For any trace-closed $K \in \mathrm{REG}$, *it holds that* $\mathrm{ext}(K_I) = [\mathrm{ext}(K)]_{\sim_I}$

I-suffix extension suffices

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For any trace-closed $K \in \mathrm{REG}$, *it holds that* $\mathrm{ext}(K_I) = [\mathrm{ext}(K)]_{\sim_I}$

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A trace-closed language $L \subseteq \Sigma^\omega$ *is recognized by an* I*-diamond DWA iff* $L \in \mathrm{BC}(\mathrm{ext}(\mathcal{K}))$ *for a set* $\mathcal{K} \subseteq \mathrm{REG}$ *of trace-closed regular languages.*

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Definition (Weakly recognizable trace languages)

A trace language $\Theta \subseteq \mathbb{R}(\Sigma^\omega, I)$ is **weakly recognizable** if $\Gamma^{-1}(\Theta)$ is I-diamond DWA recognizable.

Infinitary limits of regular trace languages

Definition

Let $T \in \mathrm{Rec}(\mathbb{M}(\Sigma^*, I))$, the **infinitary limit** $\lim(T)$ is the ω-trace language containing all $\theta \in \mathbb{R}(\Sigma^\omega, I)$ such that there exists a sequence $(t_i)_{i\in\mathbb{N}}, t_i \in T$ satisfying $t_i \sqsubset t_{i+1}$ and $\bigsqcup_{i\in\mathbb{N}} t_i = \theta$.

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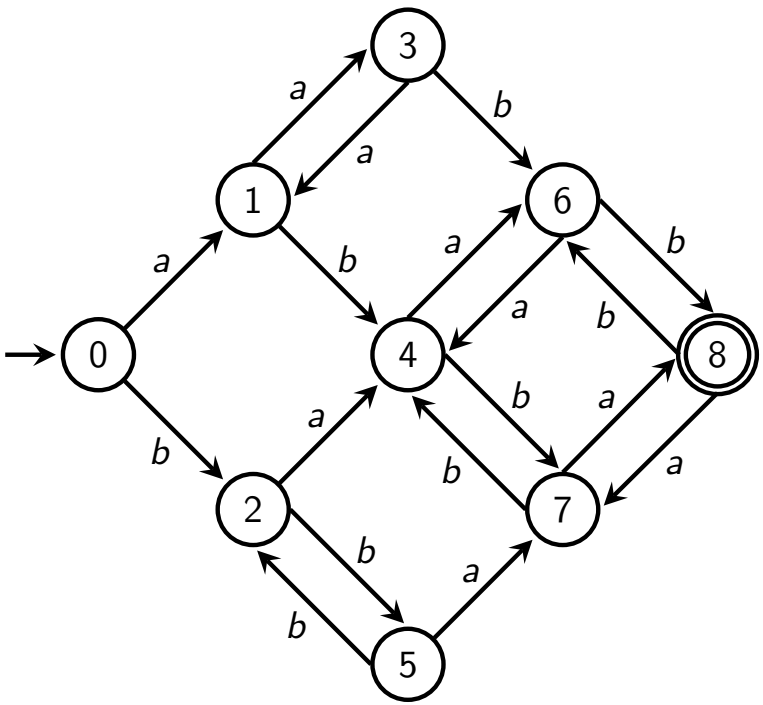
Definition (Limit-stability)

A trace-closed language $K \in \mathrm{REG}$ is **limit-stable** if $\lim(K) = [\lim(K)]_{\sim_I}$.
By extension, we say that $T \subseteq \mathbb{M}(\Sigma^*, I)$ is **limit-stable** if $\Gamma^{-1}(T)$ is.

Searching for limit-stable languages



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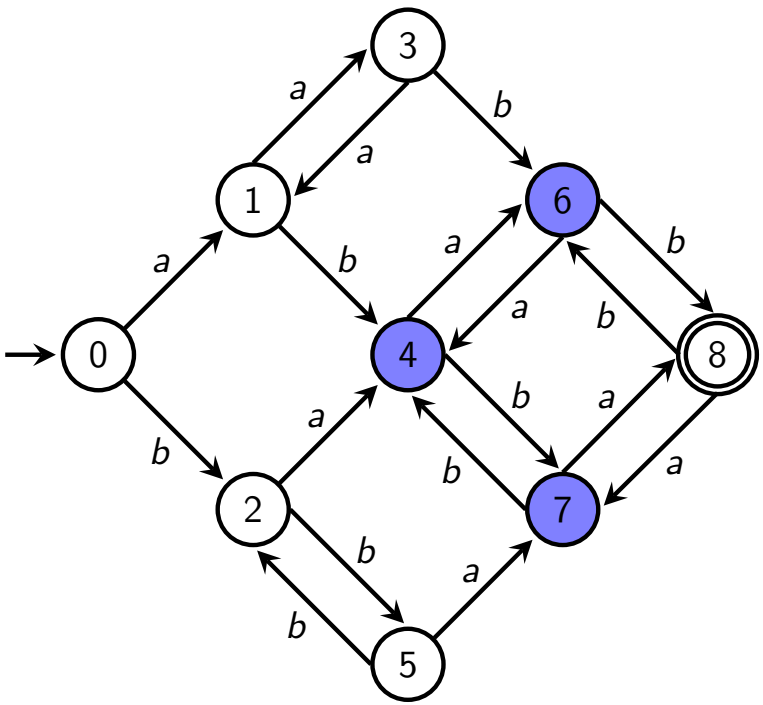

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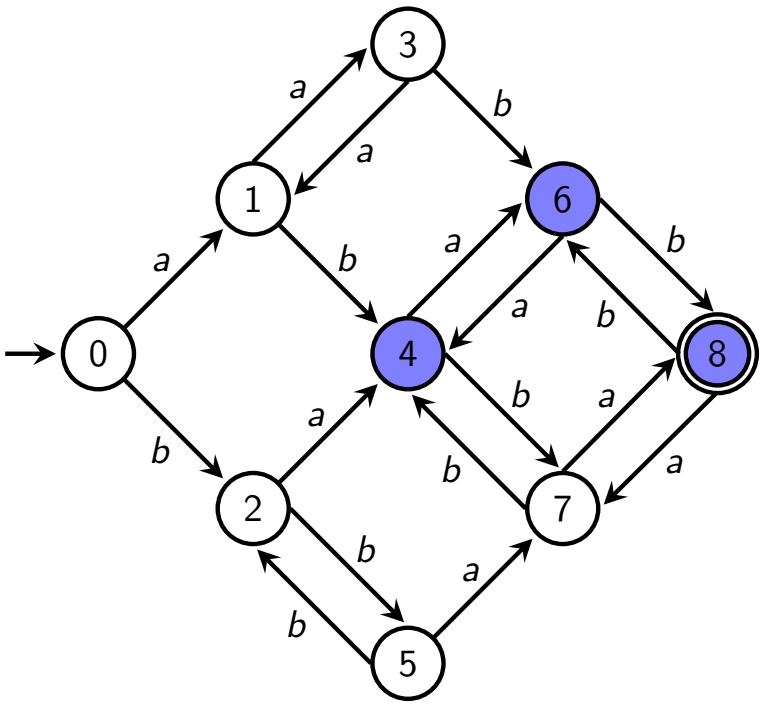

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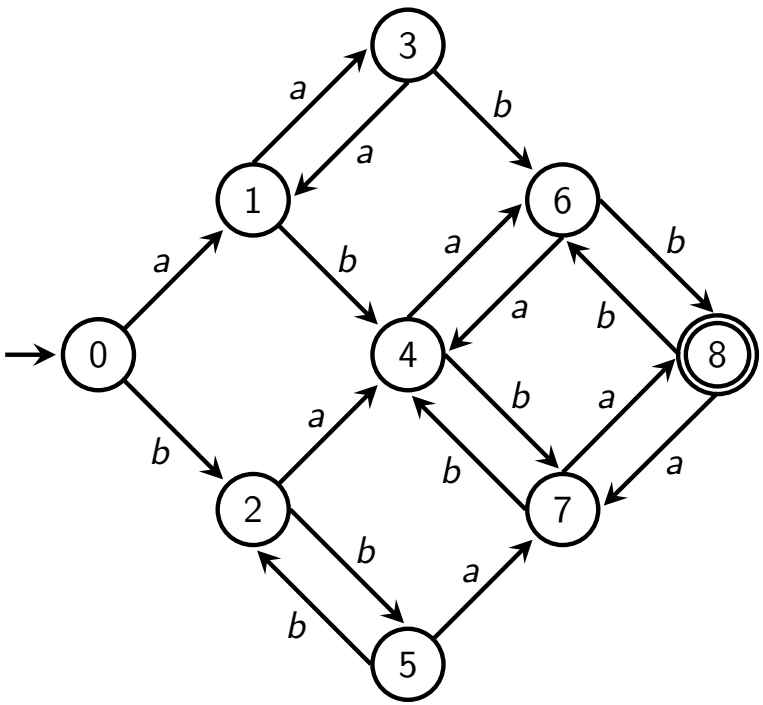
Searching for limit-stable languages

Definition (F, I-cycle closure)

For a given (Σ, I), an I-diamond DFA is F, I-**cycle closed** if for all $u \sim_I v$ and all $q \in Q$, $q \xrightarrow{u}_F q$ iff $q \xrightarrow{v}_F q$.



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A language $K \in \mathrm{REG}$ is limit-stable if and only if K is recognized by an F, I-cycle closed DFA.

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- An ω language recognized by a DBA is trace-closed if and only if the DBA is F, I-cycle closed

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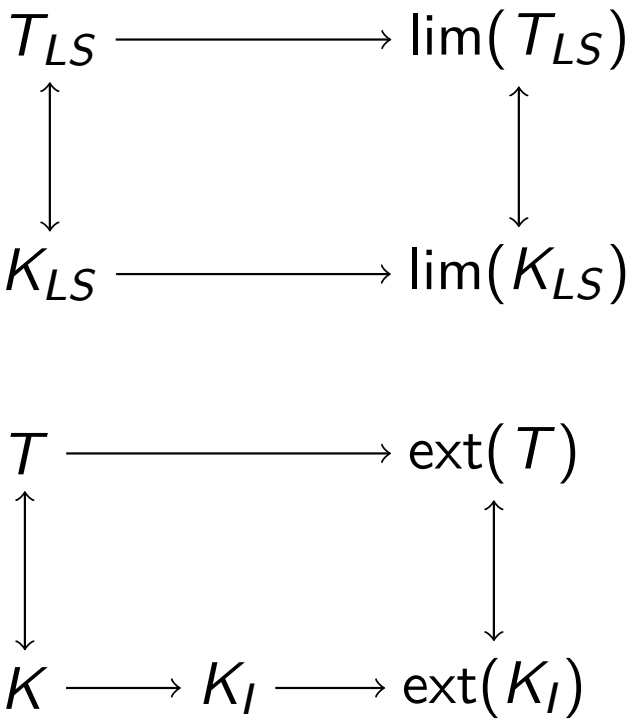
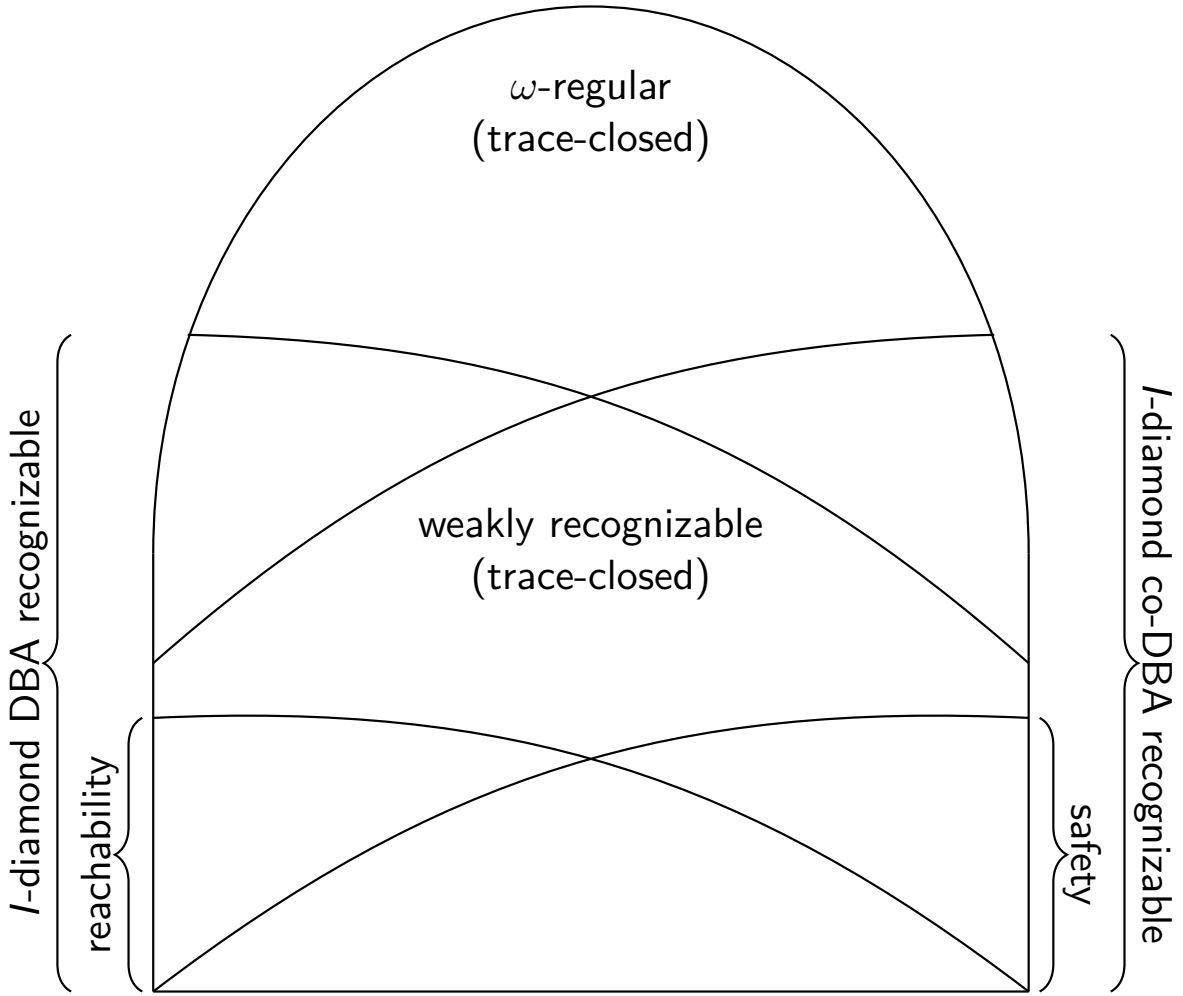
Theorem (ω-regular trace languages)

A trace language Θ is ω-regular if and only if $\Gamma^{-1}(\Theta)$ is a finite Boolean combination of I-diamond DBA recognizable trace-closed languages. That is, $\mathrm{Rec}(\mathbb{R}(\Sigma^\omega, I)) = \mathrm{BC}(\lim(LS\text{-}\mathbb{M}(\Sigma^*, I)))$.

Borel hierarchy of ω-regular trace languages





Distributed Church's Synthesis Problem

Adversary's choice of independence relation:

- $I_1 \subseteq \Sigma_1 \times \Sigma_1$ and infer $I \subseteq \Sigma \times \Sigma$ therefrom: $(a, x)\, I\, (b, y) \Leftrightarrow a I_1 b$

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A **play** of the distributed game over $\Sigma \subseteq \Sigma_1 \times \Sigma_2$ is a trace $\theta \in \mathbb{R}(\Sigma^\omega, I)$:

$$\theta = \underbrace{\begin{matrix} (a, x_a) & \searrow \\ (b, x_b) & \longrightarrow & (d, x_d) \\ (c, x_c) & \nearrow \end{matrix}}_{t}$$

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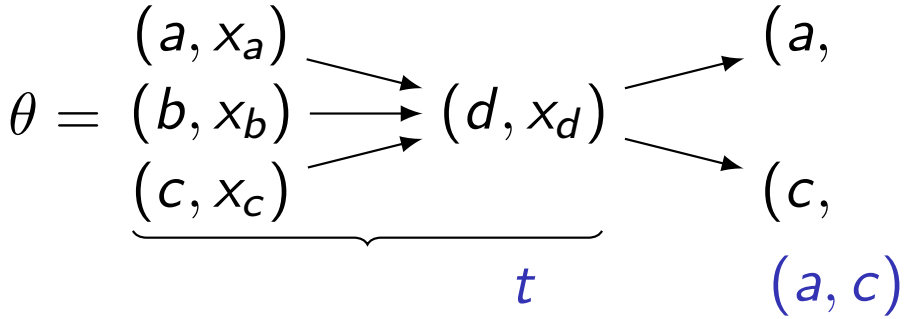

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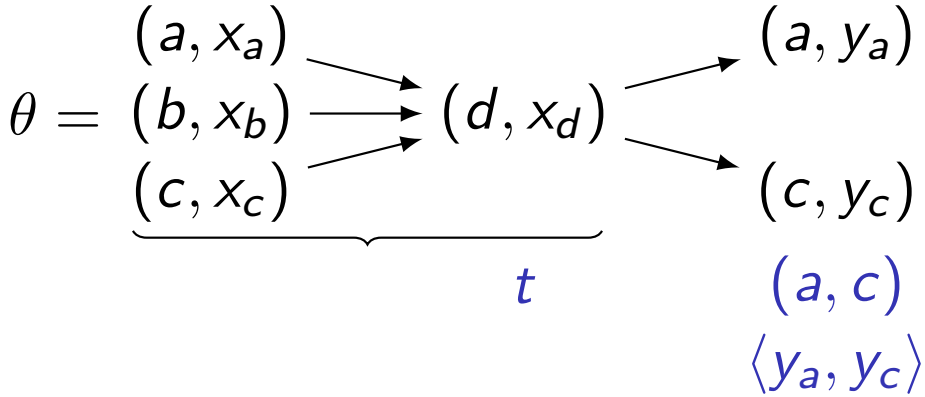

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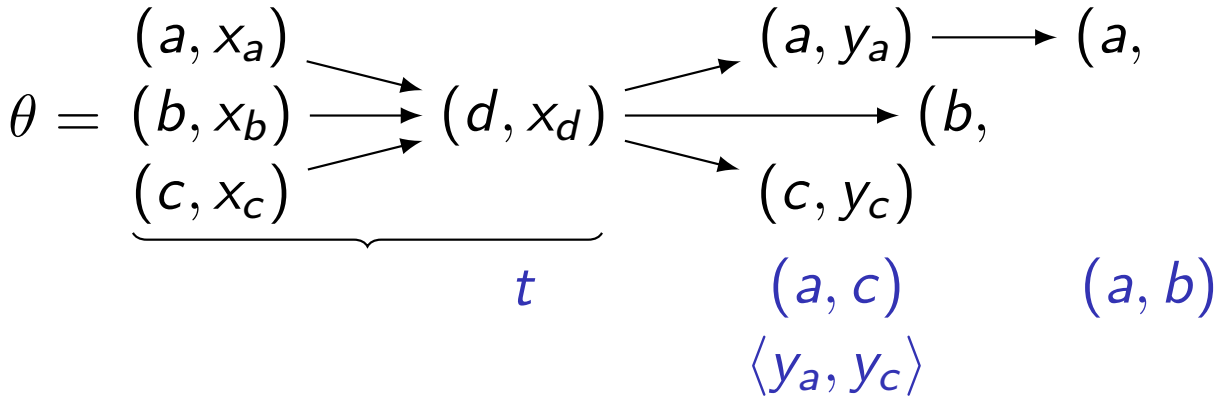

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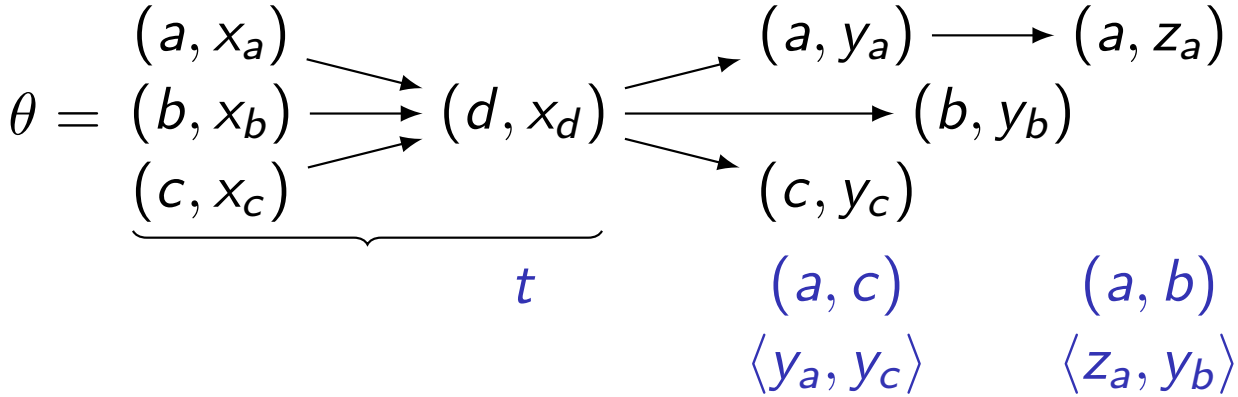

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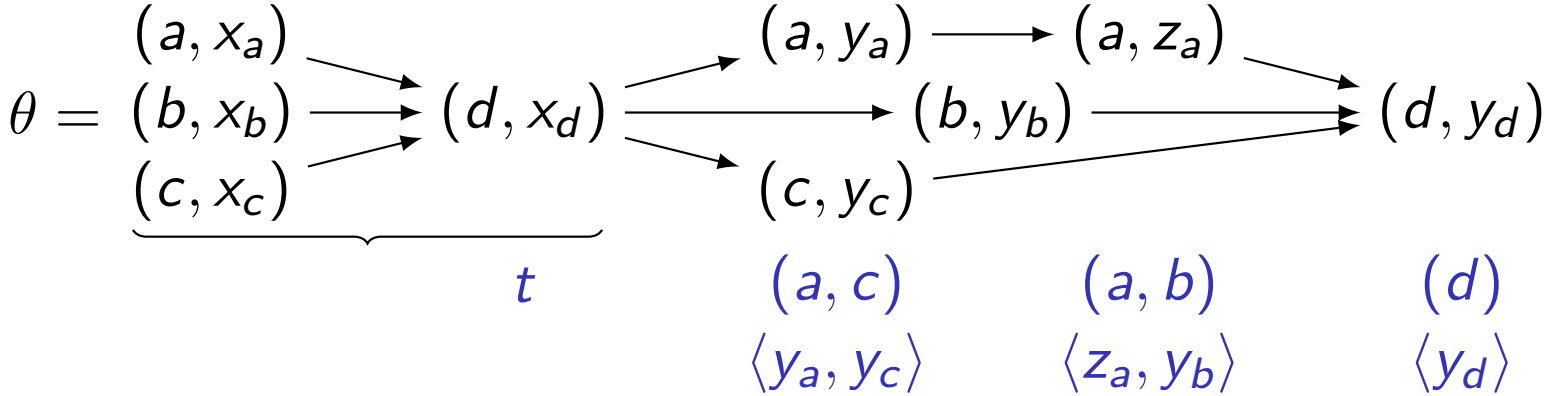

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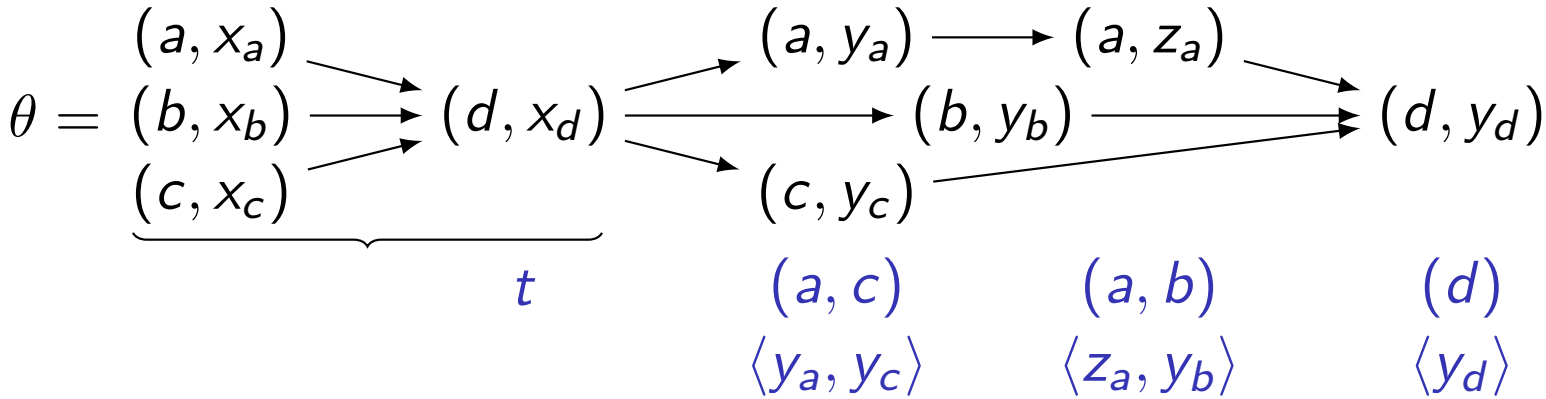

Player 1 moves (assuming Σ_1 is ordered):

- $\mathcal{C}_I := \{(a_1, \ldots, a_k) \subseteq 2^{\Sigma_1} \mid a_1 < \cdots < a_k \text{ and } a_i I a_j, i \neq j\}$

Player 2 moves:

- $\mathcal{V} := \{\langle x_1, \ldots, x_k \rangle \mid x_i \in \Sigma_2, k \leq |\Sigma_1|\}$

Distributed Church's Synthesis Problem

Game: an ω-trace language $\Theta \subseteq \mathbb{R}(\Sigma^\omega, I)$

- If $\theta \in \Theta$ then Player 2 wins
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Given finite play prefixes, **strategies** guide players:

- Strategy for Player 1: $\lambda_1 : \mathbb{M}(\Sigma^*, I) \to \mathcal{C}_I$
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To solve game $\Theta \in \mathrm{Rec}(\mathbb{R}(\Sigma^\omega, I))$, solve game $\Gamma^{-1}(\Theta)$.

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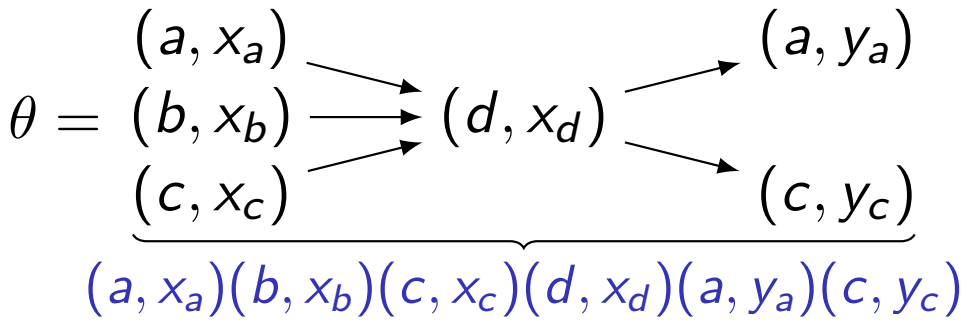
A **sequential strategy** of Player 2 is a transition system where each state $q \in Q$ has exactly one outgoing transition $(a, x_a) \in \Sigma$ for each $a \in \Sigma_1$.

Distributable sequential strategy for Player 2



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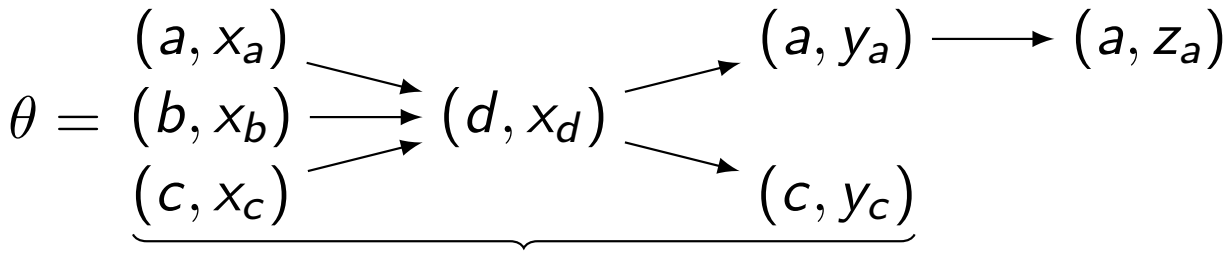

$(a, x_a)(b, x_b)(c, x_c)(d, x_d)(a, y_a)(c, y_c)$ $\quad (a)\langle z_a \rangle$

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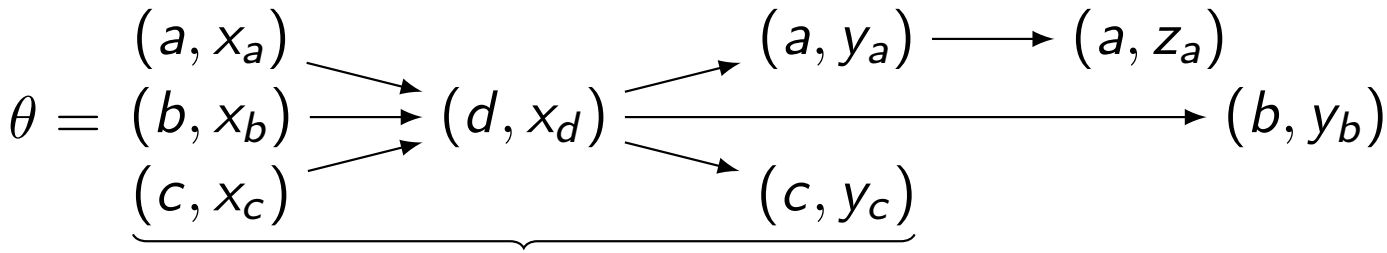

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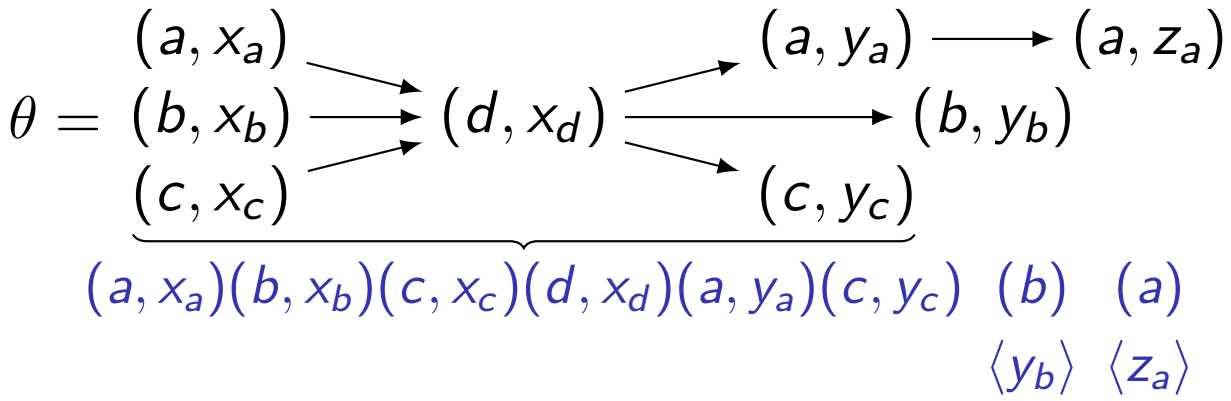

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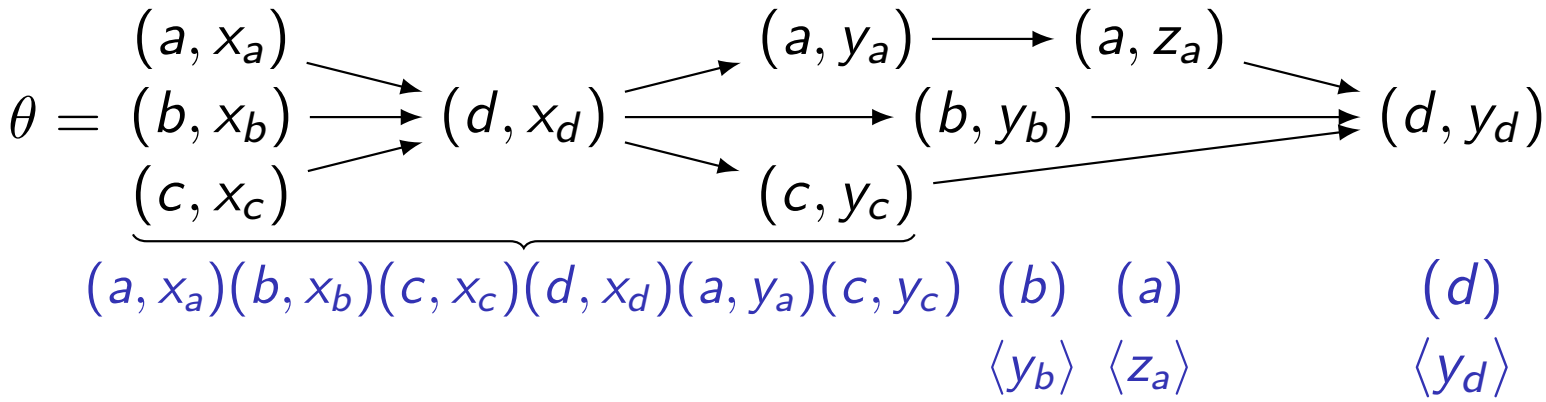

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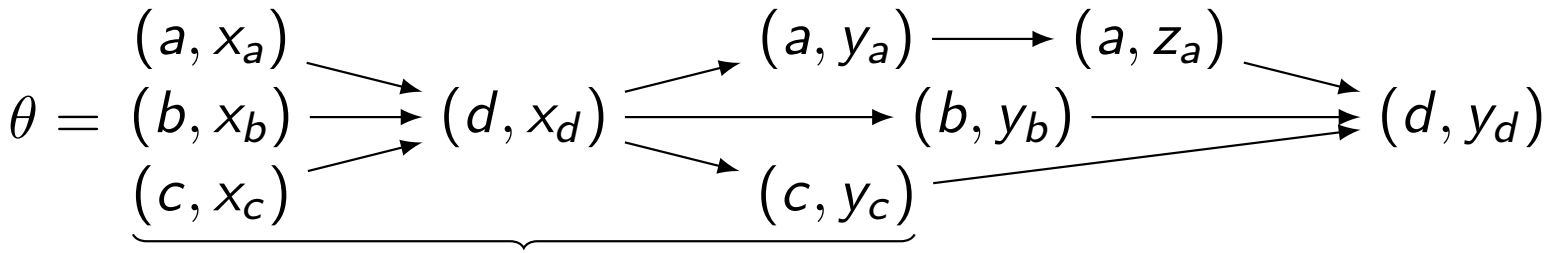

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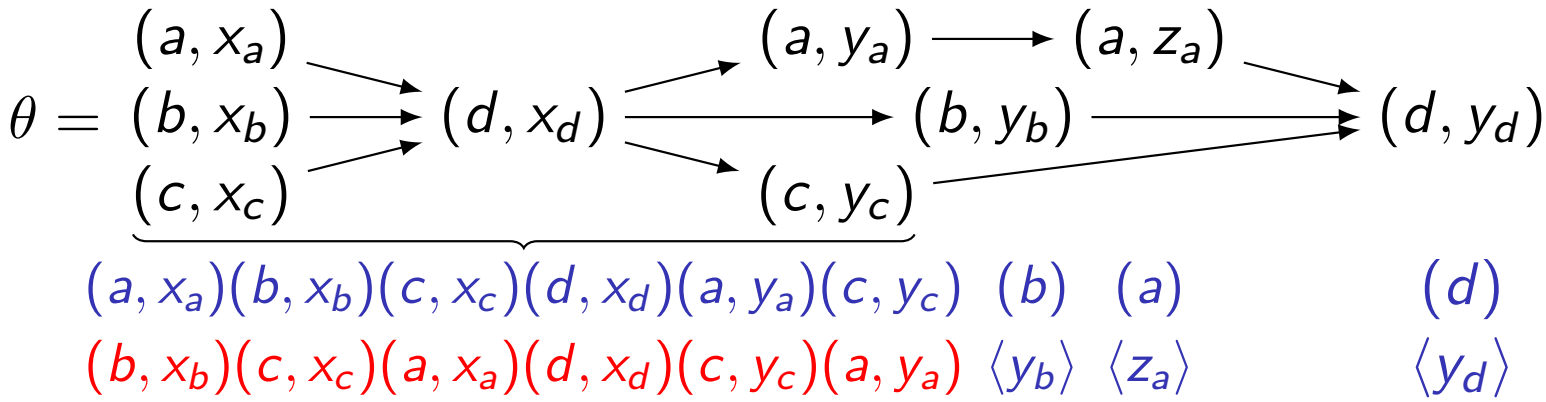

Theorem (Ștefănescu-Esparza-Muscholl, Mukund)

*There exists an **distributed strategy** for Player 2 iff there exists a **safe-branching* sequential strategy** for Player 2.*

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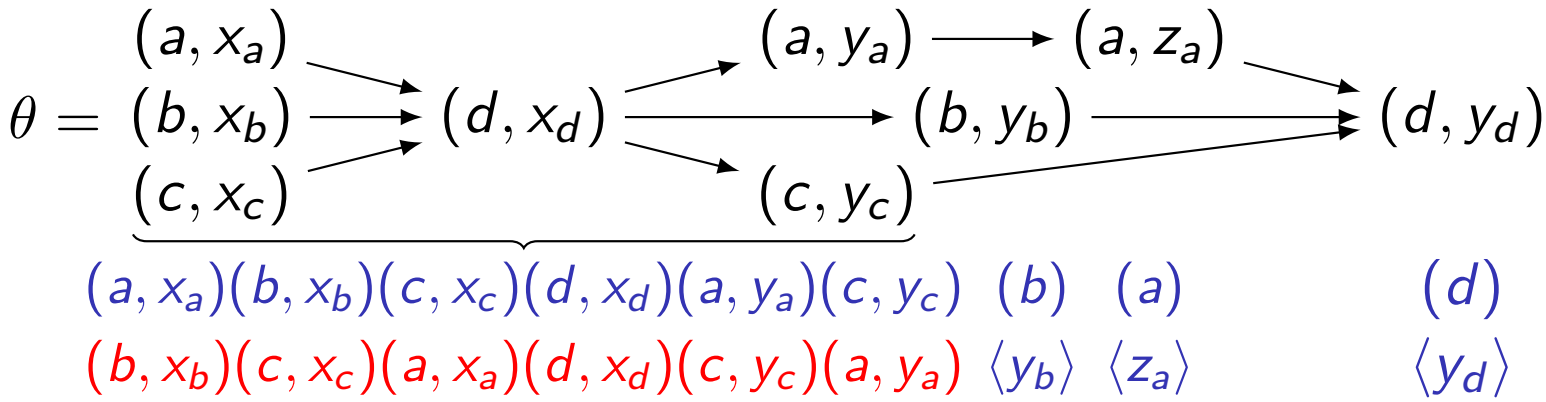

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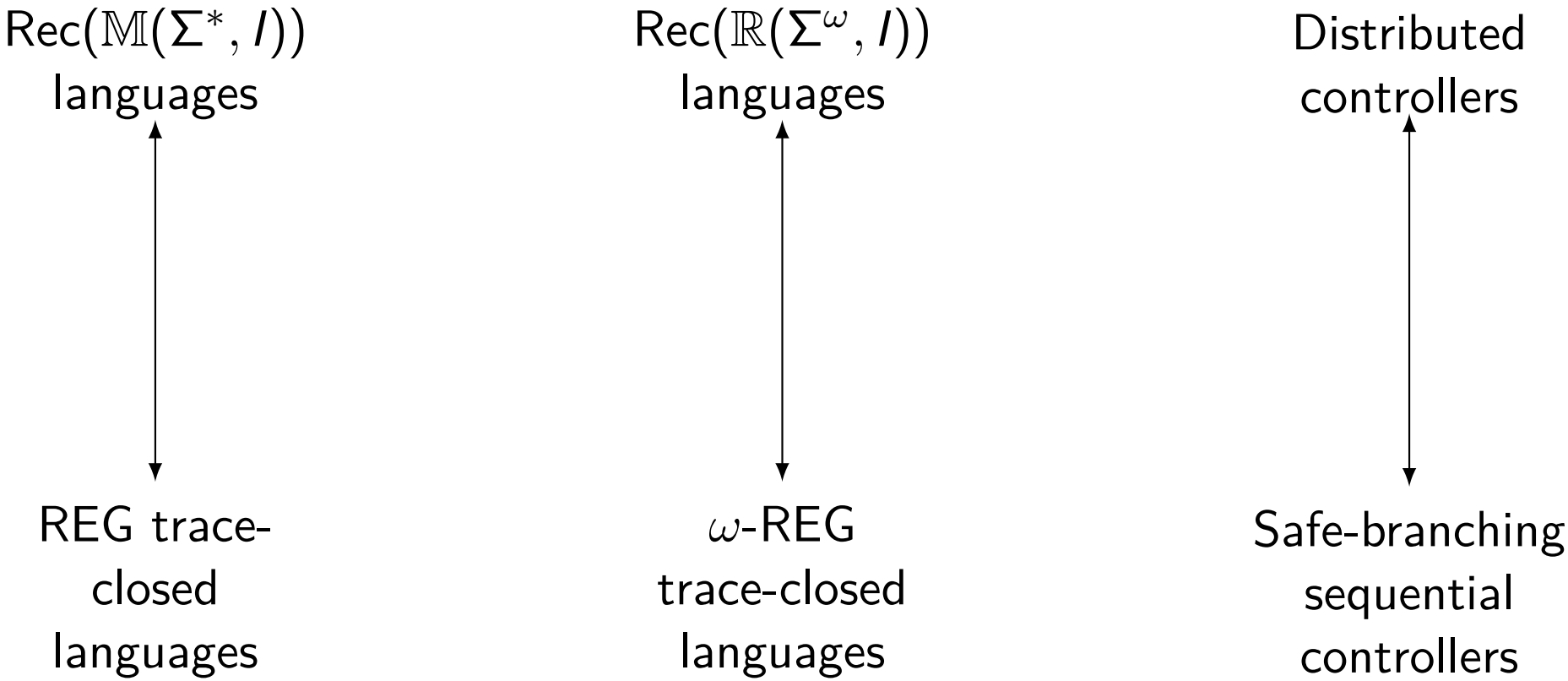
Ongoing work

Complete procedures for constructing safe-branching sequential strategies for various classes of trace-closed games.

Summary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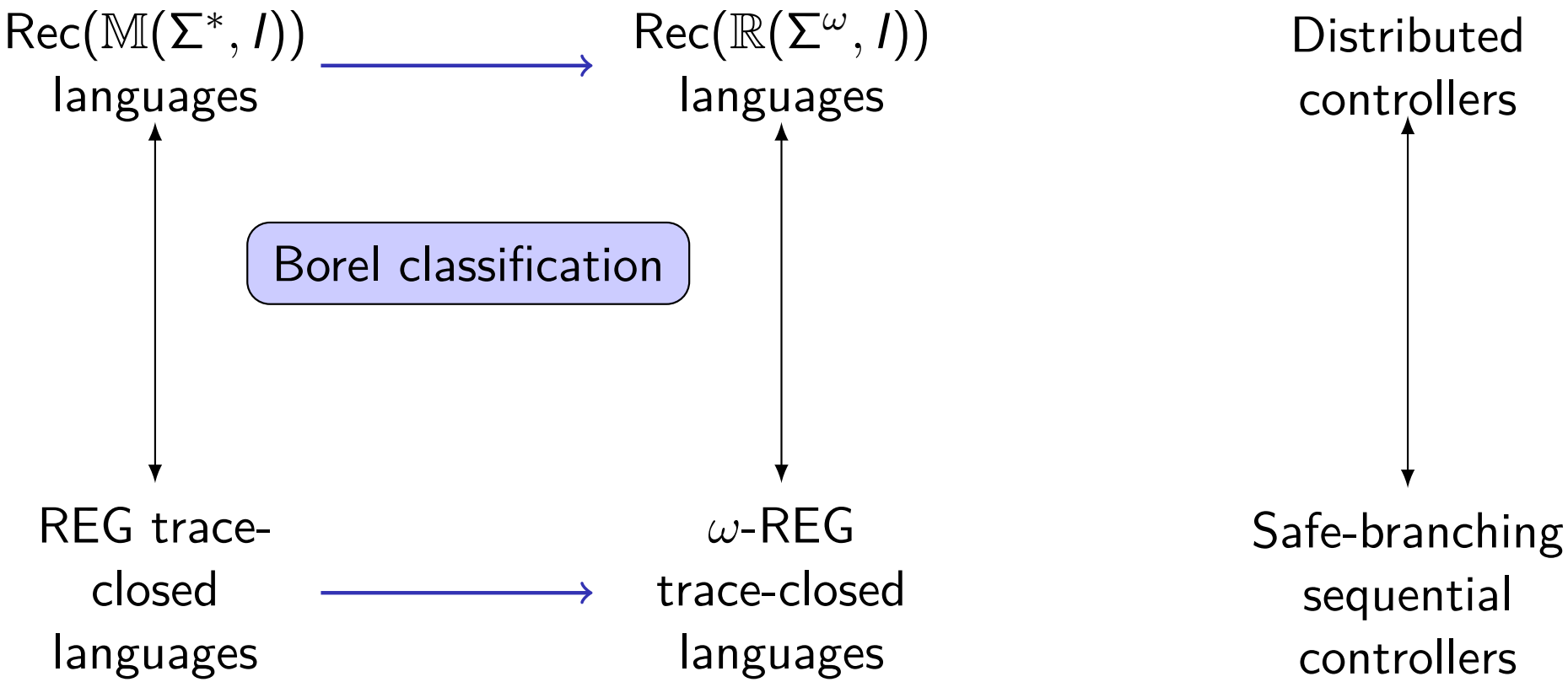
Distributed controllers
Safe-branching sequential controllers

Summary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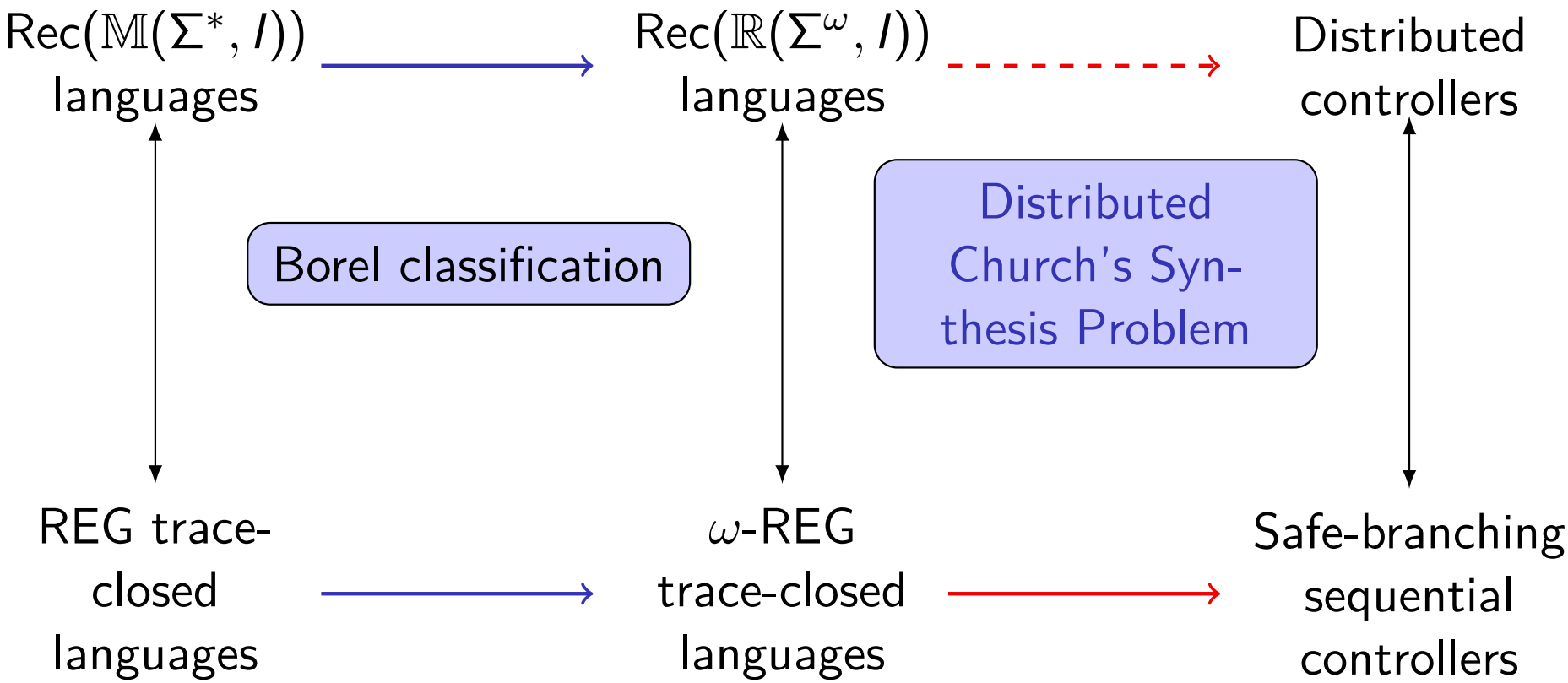
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