Path Logics with Synchronization

Wolfgang Thomas

RWTH Aachen University, Informatik 7, 52056 Aachen, Germany
thomas@informatik.rwth-aachen.de

Abstract. Over trees and partial orders, chain logic and path logic are systems of monadic second-order logic in which second-order quantification is applied to paths and to chains (i.e., subsets of paths), respectively; accordingly we speak of the path theory and the chain theory of a structure. We present some known and some new results on decidability of the path theory and chain theory of structures that are enhanced by features of synchronization between paths. We start with the infinite two-dimensional grid for which the finite-path theory is shown to be undecidable. Then we consider the infinite binary tree expanded by the binary "equal level predicate" E. We recall the (known) decidability of the chain theory of a regular tree with the predicate E and observe that this does not extend to algebraic trees. Finally, we study refined models in which the time axis (represented by the sequence of tree levels) or the tree levels themselves are supplied with additional structure.

1 Introduction

In the verification of distributed systems with nonterminating behaviour, one has to consider all possible computation paths (or "system runs"), and synchronization is captured by a merge or a connection between certain points in different computation paths.

Usually the computation paths are considered as parts of a tree structure in which all paths are collected. For the model of infinite tree (where the branching may be finite or infinite) a powerful theory of verification is available for specifications that are formalizable in MSO-logic (monadic second-order logic). Starting from Rabin's Tree Theorem [9], many beautiful and strong decidability results have been obtained, in particular on numerous branching time logics (like CTL, CTL*, ECTL*).

Most branching time logics can be expressed already in proper fragments of MSO-logic in which set quantification only ranges over paths or chains. (By a chain we mean a subset of a path.) We call these systems path logic and chain logic, respectively. If we restrict even to quantifiers over finite paths or chains, we obtain finite-path logic and finite-chain logic, respectively. Accordingly we speak of the path theory, chain theory, finite-path theory, and finite-chain theory of a structure.

Starting from chain logic (or path logic), one can gain expressive power also in other ways than by proceeding to full MSO-logic. MSO-logic typically allows to formalize conditions on a global coloring of the tree under consideration, as this is required, for instance, in a statement that a tree automaton has a successful run over the tree.

A different decidable extension of chain logic arises by adding features that capture aspects of synchronization, for example by adjoining the binary "equal level predicate" (that connects vertices of the same tree level). If full MSO-logic over the binary tree is extended by this predicate one obtains an undecidable theory (see [12]).

In the present paper we pursue the idea of extending chain logic as well as the underlying models by constructs of synchronization. For several of such extensions (which are incomparable in expressive power to MSO-logic) we clarify the status of the model-checking problem.

We study two paradigmatic versions of "synchronization" or "merging of paths". For notational simplicity we confine ourselves just to structures with binary branching. So an element has two different successors, called its 0- and 1-successor.

The first type is given by a merge of two paths if their numbers of 0- and 1-successors coincide (regardless of their order). We obtain the infinite $(\mathbb{N}\times\mathbb{N})$-grid rather than the binary tree. It is well known that the MSO-theory of the grid is undecidable. We show that even the finite-path theory of the grid is undecidable.

As a second type of synchronization, we consider the expansion of the tree by the equal level predicate E mentioned above. It holds for two points of different paths if they occur at the "same time" if time progresses discretely along each path. We recall the result of [13] that the chain theory of a regular tree with the E-predicate is decidable. A simple argument will show that this result does not extend to algebraic trees (i.e., trees that are generated by deterministic pushdown automata). Finally, we address more refined types of models in which extra distinctions enter for the axis of time (by extra predicates on the sequence of levels) and for the indivual levels themselves. We analyze under which circumstances these extra distinctions lead to an undecidability of the chain theory.

The paper is structured as follows. In the subsequent section we fix the notation. Then we address path logic over the infinite grid, and afterwards turn to chain logic over regular and algebraic trees with the equal level predicate E. Finally, we treat the question of extending the logic by refining the time axis or the individual levels of a tree. In a concluding section we discuss some related issues and open questions.

2 Structures and Logics

Two types of structures are considered in this paper, the infinite two dimensional grid (in the labelled and unlabelled version) and the infinite binary tree.

We use a format of the infinite grid that follows the idea of an infinite matrix. The positions are pairs $(i, j) \in \mathbb{N} \times \mathbb{N}$, and we have two successor functions r (right) and d (down); so formally $d(i, j) = (i + 1, j)$ and $r(i, j) = (i, j + 1)$. We write $G_2 = (\mathbb{N} \times \mathbb{N}, d, r)$. We consider labelled grids with the label alphabets

$\{0,1\}^n$, so that a labelling can be identified with an n-tuple $\overline{P} = (P_1, \dots, P_n)$ of unary predicates where P_i contains those vertices whose n-bitvector label $(b_1 \dots, b_n)$ satisfies $b_i = 1$.

The infinite binary tree is the structure $T_2 = (\{0,1\}^*, \cdot 0, \cdot 1)$ consisting of the finite bit words and equipped with the two successor functions $\cdot 0, \cdot 1$. Again, for the case of labelled trees we consider label alphabets $\{0,1\}^n$, and as above we identify a tree labelled in $\{0,1\}^n$ with a structure $T = (\{0,1\}^*, \cdot 0, \cdot 1, \overline{P})$, with a tuple $\overline{P} = (P_1, \dots, P_n)$ of unary predicates. A tree $T = (\{0,1\}^*, \cdot 0, \cdot 1, \overline{P})$ is *regular* if for each $i = 1, \dots, n$, the set P_i is regular. For notational simplicity we mostly confine ourselves to 0-1-labellings and correspondingly expansions of T_2 by a single predicate. So a tree $T = (\{0,1\}^*, \cdot 0, \cdot 1, P)$ is regular if for some finite automaton the state reached after processing w is accepting iff $w \in P$.

We also expand binary trees with the "equal level predicate" E defined by

$$E(u, v) \quad :\Leftrightarrow \quad |u| = |v|$$

We call a structure $T = (\{0,1\}^*, \cdot 0, \cdot 1, E, \overline{P})$, short $(T_2, E, \overline{P})$, a "tree with E".

For both kinds of structures (grids and trees) there is a natural notion of *path*, as a finite or infinite sequence $v_0, v_1, \dots$ (respectively $v_0, v_1, \dots, v_k$) such that v_0 is the root and v_{i+1} is a successor of v_i. By root we mean the element $(0,0)$ of the grid, respectively ε in the binary tree, and "successor" refers to the respective functions d, r, $\cdot 0, \cdot 1$. A *chain* is a subset of some path; in terms of the partial order that is generated by the successor functions this means that chains are linearly ordered subsets of the universe.

Let us introduce the logics considered in this paper. MSO-logic (monadic second-order logic) has variables $x, y, z, \dots$ for elements and variables $X, Y, Z, \dots$ for subsets of the structure under consideration. In structures where the notions of path and of chain are meaningful (as is the case over the grid and the binary tree) we can restrict the second-order quantifications accordingly. If set quantifiers range only over paths, respectively chains, then we call the resulting system *path logic*, respectively *chain logic*. In the corresponding "weak" logics we restrict even further, namely to finite paths and chains, respectively. We call these systems finite-path logic and finite-chain logic.

If S is a structure, then the MSO-theory of S is the set of MSO-sentences (of appropriate signature) that are true in S. Similarly, referring to chain logic, path logic, and their "finite" restrictions, one defines the chain theory, path theory, finite-chain theory, and finite-path theory of S.

In some situations below it will be convenient to replace MSO-logic and chain logic with an expressively equivalent variant. Two modifications are assumed: First we convert our structures to relational ones; this means that we work with the successor relations S_d, S_r in place of the functions d, r and with the relations S_0, S_1 corresponding to the successor functions $\cdot 0, \cdot 1$. Second, we replace the first-order variables by second-order varibales ranging over singletons; this helps to avoid syntactic complications. So we identify MSO-logic with the logic that has the following atomic formulas:

- $X \subseteq Y$ ("X is a subset of Y"),

– Sing(X) ("X is a singleton"),
– $X \subseteq P$ for each unary predicate P in the signature,
– $X\ R\ Y$ for each binary relation R in the signature, including equality ("X, Y are singletons and we have xRy for their elements x, y")

3 Path Logic over the Grid

In the study of monadic logic, the infinite two-dimensional grid is prominent as a simple structure whose MSO-theory is undecidable. The standard proof makes use of the possibility to describe global colorings (or labellings) of a structure in MSO-logic; even existential MSO-logic suffices for this purpose. A computation of a Turing machine M on its (left-bounded and right-infinite) tape can be represented by such a coloring of the grid, just by filling the rows of the grid successively with labellings that code the configurations of the computation. For a given Turing machine M it is easy to write down an existential MSO-sentence φ_M which describes a labelling that codes a halting computation of M when started on the empty tape. This shows that the (existential) MSO-theory of the grid G_2 is undecidable.

By another straightforward idea of coding we show that even the finite-path theory of G_2 is undecidable. We use a reduction to the termination problem of 2-counter machines (or 2-register machines). Such a machine M is given by a finite sequence

$$1 \text{ instr}_1; \ldots ; k-1 \text{ instr}_{k-1}; k \text{ stop}$$

where each instruction instr_j is of the form

– $\text{Inc}(X_1)$, $\text{Inc}(X_2)$ (increment the value of X_1, respectively X_2 by 1), or
– $\text{Dec}(X_1)$, $\text{Dec}(X_2)$ (similarly for decrement by 1, with the convention that a decrement of 0 is 0), or
– If $X_i = 0$ goto ℓ_1 else to ℓ_2 (where $i = 1, 2$ and $1 \leq \ell_1, \ell_2 \leq k$, with the natural interpretation).

An M-configuration is a triple (ℓ, m, n), indicating that the ℓ-th instruction is to be executed and the values of X_1, X_2 are m, n, respectively. A terminating M-computation (for M as above) is a sequence $(\ell_0, m_0, n_0), \ldots, (\ell_r, m_r, n_r)$ of M-configurations where in each step the update is done according to the instructions in M and the last instruction is the stop-instruction (formally: $\ell_r = k$). The termination problem for 2-counter machines asks to decide, for any given 2-counter machine M, whether there exists a terminating M-computation that starts with $(1, 0, 0)$ (abbreviated as $M : (1, 0, 0) \rightarrow \text{stop}$). It is well-known that the termination problem for 2-counter machines is undecidable.

Theorem 1. *The finite-path theory of the infinite grid G_2 is undecidable.*

Proof. For any 2-register machine M we construct a finite-path formula φ_M such that $M : (1, 0, 0) \rightarrow \text{stop}$ iff $G_2 \models \varphi_M$.

The idea is to code a computation $(\ell_0, m_0, n_0), \ldots, (\ell_r, m_r, n_r)$ by three finite paths that basically proceed along the diagonal; a path signals value i by a deviation $r^i d^i$ from the diagonal to the right and then down by i steps. (The vertex reached after applying r^i before turning down is henceforth also called the "corner vertex"; in the figure below these corner vertices are indicated by bullets.) More precisely, we code a configuration (ℓ, m, n) by three path segments S_ℓ, S_m, S_n of length $2N + 2$ each, where $N = \max(\ell, m, n)$:

$$S_\ell = drr^\ell d^\ell (dr)^{N-\ell}, \quad S_m = drr^m d^m (dr)^{N-m}, \quad S_n = drr^n d^n (dr)^{N-n}$$

All three path segments start at the diagonal, say at vertex (j, j), and return to it at vertex $(j + N + 2, j + N + 2)$ (see Fig. 1 below for an illustration).

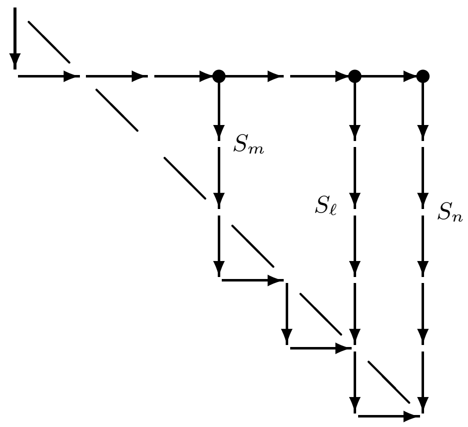

Fig. 1. Coding configuration $(\ell, m, n) = (4, 2, 5)$.

The concatenation of the these triples of path segments for the configurations $(\ell_0, m_0, n_0), \ldots, (\ell_r, m_r, n_r)$ yields three finite paths P_0, P_1, P_2 which start at vertex $(0, 0)$ of the grid.

The desired formula φ_M expresses the existence of three finite paths P_0, P_1, P_2 that code a terminating computation of M. The formulation is a straightforward exercise once the following remarks are taken into account:

1. Vertex x is on the diagonal iff it can be reached from the origin $(0, 0)$ by a path $(dr)^*$; in other words: Each finite path P containing x and such that $r(d(y)) \in P$ implies $y \in P$ must contain the origin. This is expressible in finite-path logic; we use this idea also in analogous situations when saying that two points are "on the same diagonal".
2. The start of the code of a configuration is characterized by a path segment drr of P_0.

3. That two successive triples of path segments are in conformance with the instructions of M is expressibe by the following observations: The correct update of the instruction number is expressible by fixing the corresponding path segments S_ℓ explicitly. The correct update of the values m, n of the two M-variables X_1, X_2 is possible by checking whether the "corner vertices" of the successive path segments fit: The second corner vertex is on the same diagonal of S_i as the first corner vertex if the X_i-value stays the same, and it is located one vertex left, respectively right, of the diagonal through the first vertex if we have a decrement by 1, respectively increment by 1.
4. The condition that a computation is terminating is captured by the existence of a path segment $drr^k d^k$ of P_0; this can be expressed in finite-path logic.

Trivially this undecidability result extends to path logic, finite-chain logic, and chain logic over the grid.

Another version of this result refers to satisfiability of path formulas in unlabelled finite grids (rather than satisfaction in the infinite grid). For this, one extends the formula φ_M of the proof above by the clause that the paths P_0, P_1, P_2 have to end by segments S_ℓ, S_m, S_n that code a terminating configuration. The modified formula φ'_M is satisfiable in a quadradic unlabelled finite grid iff M terminates when starting in configuration $(1, 0, 0)$. Hence we obbtain the following variant of Theorem 1:

Corollary 1. *Satisfiability of (finite-) path formulas over quadratic unlabelled finite grids is undecidable.*

4 Model-Checking over Trees with Equal Level Predicate

In this section we study the model-checking problem with respect to chain logic over structures (T_2, E, P) that are derived from the binary tree T_2 by two expansion steps: We add the equal level predicate E, capturing synchronization of vertices on the paths, and we add a unary predicate P corresponding to a labelling of the vertices by the values 0 and 1. In general, we can admit an n-tuple of predicates and thus labels from a larger alphabet $\{0,1\}^n$; we stay with the case $n = 1$ for ease of notation. We consider chain logic over a structure (T_2, E, P). Let us recall a result from [13].

Theorem 2. *The chain theory of a regular binary tree with equal level predicate is decidable.*

Proof. Let (T_2, E, P) be a regular tree with equal level predicate. We claim that there is a method to decide for any chain-sentence φ whether $(T_2, E, P) \models \varphi$.

For this, we refer to the variant of chain logic in which first-order variables are eliminated and the atomic formulas are of the form $X \subseteq Y$, $X \subseteq P$, and XEY (see Section 2). We shall provide a translation into the MSO-theory of $(\mathbb{N}, +1)$, also called Büchi's arithmetic S1S [1].

As a preparation we associate with each chain C in (T_2, E, P) a pair (α_C, β_C) of ω-words over $\{0,1\}$. The sequence α_C codes the leftmost path P_C of which C

is a subset. (If C is infinite, then this path is unique; if C is finite, we agree on the leftmost branching after the last element of C to obtain uniqueness.) So $\alpha_C = d_0 d_1 d_2 \dots$ is a sequence of "directions". The sequence β_C codes membership of elements in C along the path P_C: We have $\beta_C(i) = 1$ iff $d_0 \dots d_{i-1} \in C$ (for $i = 0$ this means $\varepsilon \in C$). In order to code membership of P_C-vertices in the given regular set P, we also introduce a third sequence γ_C by setting $\gamma_C(i) = 1$ iff $d_0 \dots d_{i-1} \in P$.

Let us identify the sequences $\alpha_C, \beta_C, \gamma_C$ with the corresponding sets of natural numbers. Using the correspondence between chains C and pairs (α_C, β_C) of 0-1-sequences and the coding of P along the path P_C by the sequence γ_C, we can rewrite any chain formula $\varphi(X_1, \dots, X_n)$ speaking about the structure (T_2, E, P) as an S1S-formula $\varphi'(X_1, Y_1, Z_1 \dots, X_n, Y_n, Z_n)$ such that

$$\begin{aligned}&(T_2, E, P) \models \varphi[C_1, \dots, C_n]\\ &\text{iff } (\mathbb{N}, +1) \models \varphi'[\alpha_{C_1}, \beta_{C_1}, \gamma_{C_1}, \dots, \alpha_{C_n}, \beta_{C_n}, \gamma_{C_n}]\end{aligned}$$

Now one observes that the reference to the γ_{C_i} is superfluous since they are (S1S-) definable from the α_{C_i}, using the fact that P is regular. Let us write $\varphi''(X_1, Y_1, \dots, X_n, Y_n)$ for the resulting S1S-formula. Applying the transformation from φ to φ'' to sentences, we obtain a reduction of the chain theory of (T_2, E, P) to the MSO-theory of $(\mathbb{N}, +1)$ (which is decidable [1]).

A natural next step after showing that the chain theory of regular trees (T_2, E, P) is decidable would be to consider algebraic trees. A tree structure (T_2, E, P) is algebraic if P can be generated by a deterministic pushdown automaton with output: After processing the word w, the pushdown automaton gives output 1 iff $w \in P$. A simple example of an algebraic tree is given by the context-free set $P = \{0^i 1^i \mid i \geq 0\}$. It is well-known that the MSO-theory of an algebraic tree (T_2, P) without the E-predicate is decidable [3]. Here we are interested in the chain theory over algebraic trees expanded by the equal level predicate.

Theorem 3. *Let $P = \{0^i 1^i \mid i \geq 0\}$. The finite-chain theory of the algebraic tree (T_2, E, P) with equal level predicate is undecidable.*

Proof. We note two simple facts:

First, the leftmost path of T_2 with the function $\cdot 0$ is a copy of the successor structure $(\mathbb{N}, +1)$.

Second, let $\varphi(x, y)$ be the formalization of the following condition as a finite-chain logic formula: "x, y are on the leftmost path, and given $x \in 0^*$, there is a vertex $z \in x \cdot 1^*$ which belongs to P, and the vertex $y \in 0^*$ is on the same level as z" (see Fig. 2).

Clearly, the formula $\varphi(x, y)$ defines the double function $(x \mapsto 2x)$ on the copy of $(\mathbb{N}, +1)$ given by the leftmost path of T_2. More formally, we have

$$(T_2, E, P) \models \varphi[0^i, 0^j] \quad \text{iff} \quad 2i = j$$

So we can interpret the weak MSO-theory of $(\mathbb{N}, +1, (x \mapsto 2x))$ in the finite-chain theory of (T_2, E, P). Since the weak MSO-theory of $(\mathbb{N}, +1, (x \mapsto 2x))$ is undecidable [10], this shows the claim.

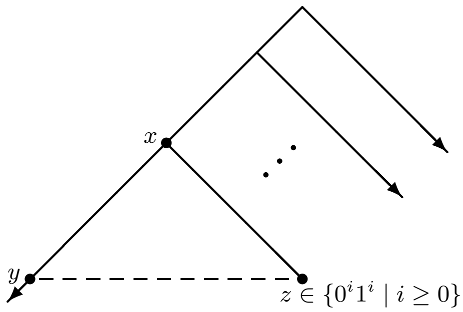

Fig. 2. Computing $y = 2x$.

5 Extra Structure

The previous result motivates to consider more modest kinds of extensions of regular tree models with E. In this section we analyze two kinds of such extensions. The first is given by extra (and of course non-regular) predicates on the time axis. Formally we represent this axis by the leftmost branch 0^ω of the tree which is identified with the set of natural numbers. For example, we may add the predicate to be a power of 2, given by the "time predicate" $Q = \{0^n \mid \exists i\ 2^i = n\}$ over the tree. We shall establish a close link between the MSO-theory of $(\mathbb{N}, +1, Q)$ and the chain theory of a structure (T_2, E, P, Q) where (T_2, E, P) is regular. (The results extend in the obvious way to the case where we consider an n-tuple $\overline{Q}$ of subsets $Q_i \subseteq 0^*$ instead of a single set $Q \subseteq 0^*$.) The second type of extension refers to additional expressive power over the individual levels of the tree structure.

5.1 Predicates on the time axis

First, let us remark that there are recursive predicates $Q \subseteq \mathbb{N}$ such that the MSO-theory of $(\mathbb{N}, +1, Q)$ – and *a fortiori* the chain theory of (T_2, E, Q) – is undecidable. To construct such a predicate Q over $\mathbb{N}$, consider a non-recursive, recursively enumerable set $R \subseteq \mathbb{N} \setminus \{0\}$ with effective enumeration $r_0, r_1, \ldots$. Define

$$Q := \{r_0, r_0 + r_1, r_0 + r_1 + r_2, \ldots\}$$

Then Q is recursive, and we have $n \in R$ iff $(\mathbb{N}, +1, Q)$ satisfies the sentence saying "there is $x \in Q$ such that the n-th successor of x is the next element in Q". This shows that even the first-order theory of $(\mathbb{N}, +1, Q)$ is undecidable.

Starting with work of Elgot and Rabin [4], many predicates Q have been found such that the MSO-theory of $(\mathbb{N}, +1, Q)$ is decidable. Prominent examples are the set of factorial numbers and, for each $k > 1$, the set of k-th powers and the set of powers of k. Further references on the subject are, for example, [2] and [11].

By a straightfoward adaptation of the proof of Theorem 2 we obtain:

Theorem 4. *Let $Q \subseteq \mathbb{N}$ and let $P \subseteq \{0,1\}^*$ be regular. The chain theory of (T_2, E, P) expanded by the time predicate Q is decidable iff the MSO-theory of $(\mathbb{N}, +1, Q)$ is decidable.*

Proof. The direction from left to right is immediate since the MSO-theory of $(\mathbb{N}, +1, Q)$ is directly interpretable in the chain theory of (T_2, E, P, Q) (considering the leftmost branch of T_2).

For the other direction, we observe that the interpretation in the proof of Theorem 2 extends to the situation where the predicate Q is added: For each chain sentence φ in the signature with E, P, Q we can construct an MSO-sentence φ' over $(\mathbb{N}, +1, Q)$ such that

$$(T_2, E, P, Q) \models \varphi \text{ iff } (\mathbb{N}, +1, Q) \models \varphi'$$

By the assumption that $\mathrm{MTh}(\mathbb{N}, +1, Q)$ is decidable the claim follows.

5.2 Quantifying over elements of levels

When a synchronization constraint refers to all existing computation paths at a certain moment, one might like to express properties of the corresponding level of the considered computation tree (T_2, E, P). Formally, we restrict to the set L of vertices at a given level and speak about a structure over the domain L. Since we are dealing with binary trees, where a natural left-to-right ordering of the paths exists, we allow the successor relation S_L and the ordering relation $<_L$ over L, and additionally unary predicates R that are chain-definable in the tree structure. More precisely, we consider a chain formula $\varphi(x)$ and introduce the corresponding predicate over L as

$$R_L^\varphi = \{v \in L \mid (T_2, E, P) \models \varphi[v]\}$$

Under which circumstances do we preserve the decidability of the chain theory of (T_2, E, P) when extending chain logic by features for expressing properties of level structures $(L, S_L, <_L, R_L^\varphi, \ldots)$?

Let us consider only two basic systems of this kind: "chain logic with E and $\mathrm{FO}(S, <)$ on the levels", and "chain logic with E and $\mathrm{MSO}(S, <)$ on the levels". In both systems we allow S and $<$ with their explained meaning (i.e., connecting vertices on a common level) and first-order, respectively monadic second-order quantifiers over elements belonging to a given level. Thus we include quantifiers $\exists y \in L_x$, $\exists X \subseteq L_x$ and the corresponding universal versions, meaning "there is a vertex y on the level of x", "there is a set X of vertices on the level of x", respectively. So, for example, in chain logic with E and $\mathrm{MSO}(S, <)$ on the levels we can express that there is a level with an even number of vertices of a certain property φ. More generally, we can express the existence of certain partitions of the paths meeting a certain level. Such conditions occur, for example, in the study on logics of knowledge studied by Halpern and Vardi [6]. We show the following result, which indicates (in part (b)) severe limitations for decidability results.

Theorem 5. *(a) The chain theory of a regular tree with E and $FO(S,<)$ on the levels is decidable.*

(b) Even the finite-path theory of the unlabelled binary tree with E and $MSO(S,<)$ on the levels is undecidable.

Proof. Part (a) is easy, since each FO($S,<$)-formula restricted to a level of the tree model is directly expressible in chain logic with the equal level predicate E.

For part (b) we use an idea of [8] that allows to code a coloring of a binary tree up to (and exluding) level L by a coloring of level L itself. We simply transfer the color of vertex v (before level L) to the unique vertex $v' \in L$ which belongs to $v10^*$ (i.e. belongs to the leftmost path from the right successor of v). The map $v \mapsto v'$ is injective and definable in finite-path logic (given level L). Moreover, it is easy to see that the relations of being left or right successor in the tree are translated to definable relations over the level L under consideration.

Using this coding, an existential quantifier over finite sets in the binary tree is captured by an existential quantifier over the elements of an appropriate level of the tree. Thus, the weak MSO-theory of the binary tree with E is interpretable in the finite-path theory of the unlabelled binary tree with E and MSO($S,<$) on the levels. Since the weak MSO-theory of (T_2, E) is undecidable (see e.g. [12]), we obtain claim (b) of the Theorem.

6 Conclusion

In this paper, we analyzed path logic and chain logic over structures that arise from the binary tree by different versions of "merging paths". Concrete examples were the infinite grid and (labelled) infinite trees expanded by the "equal level predicate" that captures a synchronization of paths. We clarified the status of the model-checking problem of path logic and chain logic over these structures and certain natural extensions.

Many issues on path logics with features of synchronization are unresolved. A natural question is to extend the present results to the richer logics of "time granularities" studied by Franceschet, Montanari, Puppis et al. in [5, 7]. Also one might refine the results in the framework of branching time logics in order to get better complexity bounds (than the nonelementary bounds as inherent in the reduction to S1S). Furthermore, it should be analyzed whether stronger theories than S1S can be invoked for interpretation, in order to show decidability for more powerful mechanisms of synchronization. Another direction is the inclusion of aspects as they are used in timed systems (where computation paths are subject to constraints on durations).

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